PROCEEDINGS

OF THE

AMERICAN SOCIETY OF CIVIL ENGINEERS

VOL. 62 FEBRUARY, 1936 NO. 2

TECHNICAL PAPERS

AND

DISCUSSIONS

Published monthly, except June and July, at 99–129 North Broadway, Albany, N. Y., by the American Society of Civil Engineers, Editorial and General Offices at 33 West Thirty-ninth Street, New York, N. Y. Reprints from this publication may be made on condition that the full title of Paper, name of Author, page reference, and date of publication by the Society, are given.

Entered as Second-Class Matter, December 28, 1931, at the Post Office at Albany, N. Y., under the Act of March 3, 1879. Acceptance for mailing at special rate of postage provided for in Section 1103, Act of October 3, 1917, authorized on July 5, 1918.

Subscription (if entered before January 1) $8.00 per annum. Price $1.00 per copy.

CURRENT PAPERS AND DISCUSSIONS

		Discussion closes
Analysis of Multiple Arches. *Alexander Hrennikoff*	Dec., 1934	
Discussion (Author's closure)	May, Sept., 1935, Feb., 1936	Closed
A Direct Method of Moment Distribution. *T. Y. Lin*	Dec., 1934	
Discussion	Mar., May, Aug., 1935	Closed
Analysis of Thick Arch Dams, Including Abutment Yield. *Philip Cravitz*	Jan., 1935	
Discussion (Author's closure)	Sept., 1935, Feb., 1936	Closed
The Hydraulic Jump in Terms of Dynamic Similarity. *Boris A. Bakhmeteff and Arthur E. Matzke*	Feb., 1935	
Discussion. (Authors' closure)	Mar., May, Aug., Sept., Nov., 1935 Feb. 1936	Closed
Frictional Resistance in Artificially Roughened Pipes. *Victor L. Streeter*	Feb., 1935	
Discussion (Author's closure)	Aug., 1935, Feb., 1936	Closed
Line Load Action on Thin Cylindrical Shells. *Herman Schorer*	Mar., 1935	
Discussion	Sept., Nov., Dec., 1935	Closed
Structural Beams in Torsion. *Inge Lyse and Bruce G. Johnston*	Apr., 1935	
Discussion (Authors' closure)	Aug., Oct., 1935, Feb., 1936	Closed
Photo-Elastic Determination of Shrinkage Stresses. *Howard G. Smits*	May, 1935	
Discussion	Sept., Oct., 1935	Feb., 1936
The Shear-Area Method. *Horace B. Compton and Clayton O. Dohrenwend*	May, 1935	
Discussion	Aug., Oct., Dec., 1935	Closed
Flood-Stage Records of the River Nile. *C. S. Jarvis*	Aug., 1935	
Discussion	Nov., Dec., 1935, Jan., 1936	Feb., 1936
Distribution of Stresses under a Foundation. *A. E. Cummings*	Aug., 1935	
Discussion	Oct., Nov., Dec., 1935, Jan., 1936	Feb., 1936
Some Low-Temperature Characteristics of Bituminous Paving Compositions. *Hugh W. Skidmore*	Aug., 1935	
Discussion	Nov., Dec., 1935	Closed
Failure Theories of Materials Subjected to Combined Stresses. *Joseph Marin*	Aug., 1935	
Discussion	Oct., Nov., Dec., 1935	Feb., 1936
Adaptation of Venturi Flumes to Flow Measurements in Conduits. *Harold K. Palmer and Fred D. Bowlus*	Sept., 1935	
Discussion	Nov., 1935, Feb. 1936	Mar., 1936
The Stress Function and Photo-Elasticity Applied to Dams. *John H. A. Brahtz*	Sept., 1935	
Discussion	Nov., Dec., 1935, Jan., Feb., 1936	Mar., 1936
Flood and Erosion Control Problems and Their Solution. *E. Courtland Eaton*	Sept., 1935	
Discussion	Nov., Dec., 1935, Feb., 1936	Mar., 1936
The Relation of Analysis to Structural Design. *Hardy Cross*	Oct., 1935	
Discussion	Dec., 1935, Jan., 1936	Mar., 1936
Tunnel and Penstock Tests at Chelan Station, Washington. *Ellery R. Fosdick*	Oct., 1935	
Discussion	Feb., 1936	Mar., 1936
Tapered Structural Members: An Analytical Treatment. *Walter H. Weiskopf and John W. Pickworth*	Oct., 1935	
Discussion	Feb., 1936	Mar., 1936
Proposed Improvements of the Cape Cod Canal. *E. C. Harwood*	Oct., 1935	
Discussion	Dec., 1935, Feb., 1936	Mar., 1936
Influence of Diversion on the Mississippi and Atchafalaya Rivers. *E. F. Salisbury*	Nov., 1935	Mar., 1936
Stable Channels in Erodible Material. *E. W. Lane*	Nov., 1935	
Discussion	Feb., 1936	Mar., 1936
Truss Deflections: The Panel Deflection Method. *Louis H. Shoemaker*	Nov., 1935	
Discussion	Jan., Feb., 1936	Mar., 1936
Lateral Pile-Loading Tests. *Lawrence B. Feagin*	Nov., 1935	
Discussion	Nov., 1935, Jan., Feb., 1936	Mar., 1936
Sedimentation in Quiescent and Turbulent Basins. *J. J. Slade Jr*	Dec., 1935	
Discussion	Feb., 1936	Apr., 1936
Wind Stresses in Reinforced Concrete Arch Bridges. *A. A. Eremin*	Dec., 1935	Apr., 1936
Successive Eliminations of Unknowns in the Slope Deflection Method. *John B. Wilbur*	Dec., 1935	Apr., 1936
Reinforced Concrete Members Under Direct Tension and Bending. *D. B. Gumensky*	Dec., 1935	Apr., 1936
Tall Building Frames Studied by Means of Mechanical Models. *Francis P. Witmer and Harry H. Bonner*	Jan., 1936	Apr., 1936
Modern Conceptions of the Mechanics of Fluid Turbulence. *Hunter Rouse*	Jan., 1936	Apr., 1936
Comparison of Sluice-Gate Discharge in Model and Prototype. *Fred William Blaisdell*	Jan., 1936	Apr., 1936

Note.—The closing dates herein published. are final except when names of prospective discussers are registered for special extension of time.

VOL. 62 PROCEEDINGS NO. 2

CONTENTS FOR FEBRUARY, 1936

PAPERS

REPORTS

DISCUSSIONS

CONTENTS FOR FEBRUARY, 1936 (*Continued*)

For Index to all Papers, the discussion of which is current in PROCEEDINGS, *see page 2*

SOCIETY AFFAIRS

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

BEHAVIOR OF STATIONARY WIRE ROPES IN TENSION AND BENDING

By Douglas M. Stewart[1], Jun. Am. Soc. C. E.

Synopsis

This experimental investigation of the behavior of wire ropes in tension and bending was undertaken in order to determine their strengths and the stresses produced under load, and to compare these values with those given by the several formulas in common use. Altogether, nine tension and thirty-six bending specimens were tested over sheaves of four diameters. The ropes selected were 1 in. in diameter, with hemp centers, and the tests included studies of regular and Lang lay ropes, of 6 × 7 and 6 × 19 construction preformed and non-preformed types, and of two different grades of steel. Important results are contained in the curves for loss of strength in bending and for the variation in modulus of elasticity of the rope under pre-stressing, and a comparative summary is given of stresses and strengths as observed and as computed by several formulas.

Introduction

Since wire ropes were first produced in the early part of the Nineteenth Century, with a view to obtaining high strength combined with flexibility over sheaves, the question of the stresses set up by bending them has been a subject of sharp controversy. Literally dozens of formulas have been developed to evaluate this bending stress, most of them of an empirical nature, and each wire rope user, in the past, has given preference to one or another in the light of his practical experience with ropes in service. In some cases, the formula was merely an expression of the results of a series of tests on specimens to determine the loss of strength over various sizes of sheaves, from which the bending stress could be evaluated in some measure. This has led recently to the expression of formulas for loss of strength in bending, which, in the end, is a more practical concept than that of the stresses to which this loss is due.

Note.—Discussion on this paper will be closed in May, 1936, *Proceedings*.

[1] Sales Engr., Ingersoll-Rand Co., New York, N. Y.; formerly Garrett Linderman Hoppes Research Fellow in Civ. Eng., Lehigh Univ., Bethlehem, Pa.

It was for the purpose of investigating the merits of these numerous bending formulas that this test program was originally conceived. Undoubtedly, there is a marked difference between the stress conditions in a stationary wire rope bent over a sheave and in one which is in rapid motion over the same sheave and possibly subjected to reverse bending as well. The scope of this investigation has been limited to a study of stationary ropes only, and while they hold admittedly a relatively minor place in wire-rope usage, the results may point the way to a clearer understanding of stress conditions in moving ropes as well as in stationary ones.

A program of tests of ropes over sheaves was planned accordingly, and a means devised for measuring the stress in any of the outer wires. For purposes of comparison a tension specimen of each type of rope was needed, and further stress observations were taken on these specimens. Because of the need in certain stress formulas for a value of the modulus of elasticity of the rope as a whole, numerous observations of this property were made, and this determination soon became one of the major branches of the investigation. Considerable data have been collected also on the untwisting effect in wire ropes under tension, on their shrinkage in diameter as their hemp centers are consolidated, and on the coefficient of friction between rope and sheave.

Notation.—The symbols used in this paper are summarized for reference in Appendix I.

The Problem

Review.—Probably the first and simplest formula that has been derived for the purpose of expressing the stress in a wire rope bent over a sheave was that of Reuleaux[2],

$$s = E \frac{d}{D} \qquad (1)$$

which was derived from the expression for bending stress in a slightly curved beam, by substituting instead of the diameter of the rod acting as a beam, the diameter of one wire (presumably in the outer layer) used in the rope. D is the diameter of the sheave and E, the modulus of elasticity of the wire, generally assumed to be about 28 500 000 lb per sq in. This formula is given by the late Robert Charles Strachan, M. Am. Soc. C. E. (4)[3], F. C. Carstarphen, M. Am. Soc. C. E. (1), and by numerous other writers.

It was soon found that Equation (1) gave values of the stress which were far too high to be practicable, in many cases even exceeding the ultimate strength of the wire for small sheave sizes. Accordingly, attempts were made to modify the formula by empirical and semi-empirical means. Shortridge Hardesty, M. Am. Soc. C. E. (2), has derived the formula (4):

$$s = E \frac{d}{D} \cos a \cos b \qquad (2)$$

[2] Given by Reuleaux in his book "The Constructor", 1893 Edition, and probably as early as 1876 in other writings.

[3] For reference to figures in parentheses, see "Bibliography", Appendix II.

in which a is the angle between a helical wire and the axis of the strand and b is the angle between a strand and the axis of the rope.

Mr. R. W. Chapman (5) modified Equation (2) by expressing the formula as:

$$s = E \frac{d}{D} \cos^2 a \cos^2 b \qquad (3)$$

which gives values for the stress lower than those given by Equations (1) and (2).

B. R. Leffler, M. Am. Soc. C. E. (3) gives an empirical modification of this formula as adopted by the New York Central Railroad Company in 1928,

$$s = \frac{2\,Ed}{3\,D} \cos^2 a \cos^2 b \qquad (4)$$

Mr. Carstarphen (1) makes mention of an empirical formula of even simpler form,

$$s = 0.44\ \mathrm{E} \frac{d}{D} \qquad (5)$$

although it is not mentioned on what test results this formula is based.

All the preceding formulas have involved the use of the modulus of elasticity of the wire, and the tendency has been to reduce the abnormally high stress values by some coefficient. In 1918, Mr. James F. Howe (2) suggested that the proper value of E to use in a formula of the general type of Equations (1) to (5) was the modulus of elasticity of the rope as a whole, E_r; thus,

$$s = E_r \frac{d}{D} \qquad (6)$$

or, as it is often used,

$$s = \frac{E_r\, d}{D + d_r} \qquad (7)$$

This formula gave values considerably lower than those of Equation (2) or Equation (3), when using a value of 12 000 000 lb per sq in. as the modulus of elasticity of the rope.

In 1933, Mr. Carstarphen (1) approached the problem from an analytical standpoint, and on the basis of a wire rope consisting of a double set of open-coiled helical springs, in turn bent around a constant radius, arrived at an expression for the loss of strength of a wire due to such bending:

$$P = \frac{\pi\,(d)^4\, E\, G}{16\, R\, r_s\, [2\, G\, (1 + \sin^2 \alpha) + E \cos^2 \alpha]} \qquad (8)$$

in which P = loss of strength in a given wire; d = the diameter of a given wire; E = modulus of elasticity of a given wire; G = modulus of rigidity of a given wire, $\left[G = \frac{E}{2\,(1 + \mu)}\right]$; μ = Poisson's ratio; R = radius of a sheave, $\left[R = \frac{1}{2}(D + d_r)\right]$; r_s = radius from the center of the strand to the center of the wire in question; and, α = the angle between the perpendicular to the axis of a rope and the tangent to the center line of the wire.

In Equation (8), when $r_s = 0$, substitute r_r which is defined as the radius from the center of a wire rope to the center wire of a strand.

The total loss of strength in a rope is equal to the value of P multiplied by the number of wires, or if the strand consists of a number of layers of different sized wires, a value of P must be computed for each layer and multiplied by the number of wires of that size in the layer. If a value of the bending stress were desired, this could presumably be obtained by dividing ΣP by the net area of steel in the rope. According to Mr. Carstarphen, Equation (8) "takes into account the diameter of the wires, the rope, the radius of curvature, the angle of lay, the modulus of elasticity in tension, and the modulus of rigidity." The test results reported in the same paper seemed to support this method of computing loss of strength.

Any of the preceding formulas, for bending stress, f, may be adopted to give loss of strength, or ultimate strength in bending, S, by inserting them in the general form of the equation:

$$S = A\ (t - s)\ \varepsilon \qquad (9)$$

in which A is the net area of steel in a wire rope; t is the ultimate unit tensile strength of a wire; and ε is the efficiency of the rope in plain tension. Equation (8) is given by C. D. Meals, Assoc. M. Am. Soc. C. E., (1) and others using Equation (7) to determine a value for f. Mr. Meals further developed an equation for the strength of a wire rope in tension, from which the efficiency, ε, might be computed:

$$S_t = n_s \cos b\ \left(\sum_{1}^{n} n_w\, S_i \cos^3 a_i\right) \qquad (10)$$

in which n_s is the number of strands in a rope; n_w is the number of wires of a given diameter in the ith layer; S_i is the tensile strength of a wire in the ith layer; a_i is the angle of pitch of the wires in the ith layer; and n_l is the number of layers of wires in a strand.

J. H. Griffith, M. Am. Soc. C. E., and Mr. J. E. Bragg (6) gave both Equation (9) and an empirical formula for tensile load based on the minimum results of tests, as:

$$T = C \times 75\,000\ d^2_r \qquad (11)$$

in which, D is the diameter of the cable, in inches; and C is a constant for various constructions (see Table 1).

TABLE 1.—Values of C for Mean in Equation (11)

Rope	Range		Mean*
	From:	To:	
6×19 plow steel	0.9	1.1	1.0
8×19 plow steel	0.8	1.00	0.85
6×19 cast steel	0.8	1.00	0.85
6×42 tiller rope	0.3	0.45	0.35
6×7 guy rope	0.3	0.45	0.35

* Approximate.

On the assumption that slipping does not occur between the straight and curved wires, Mr. Carstarphen gave the following formula (1) for the tensile strength of a rope,

$$S = \cos\,(a + b)\; A\; S_w \ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

in which S_w is the ultimate strength of the wire.

The foregoing equations for stresses and strengths in bending and in tension are only a few of the many that can be found in engineering literature. They were chosen as representative of current usage, and the range in values given by them demonstrates clearly the uncertainty that still exists as to the ultimate effect of bending stresses on the strength of a wire rope. Each of these formulas has been applied to the wire ropes used in this investigation, and a table of the results is included herein, under the heading, "Summary".

The Present Investigation.—The most logical manner in which to determine the bending stresses and loss of strength in wire ropes, seemed to be a series of tests on ropes on which the stresses could be measured by some standard extensometer. Fortunately, such equipment was available in the form of four tensometers that could be mounted on individual outer wires at different points around the sheave. From these readings unit strains were recorded directly, by multiplying by the predetermined constant for each instrument. To convert these values to unit stresses, it was necessary to draw, from auxiliary samples of the wires used, stress-strain curves for each size of outer wires encountered. From these curves the observed strains could then be transformed readily to their corresponding stress values, thus giving values of the stress in the outer wires at any point along a sheave or on a straight tension specimen.

This same principle is made use of time and time again in laboratory work on mild steel specimens, where extensometer readings of strain below the elastic limit are multiplied by 29 000 000 lb per sq in. in order to give the stress at these points. The difference lies in the fact that steel, such as is used in wire-rope manufacture, does not have a sharp, well-defined elastic limit since it is heat-treated, with the result that the stress-strain curve shows a proportional limit of about 40% of the ultimate strength. Furthermore, in the tests to destruction, strains were recorded on the ropes in most cases to about 90% of the ultimate load, the uniformity of the stress-strain curves of the wire even at these high loads permitting such readings to be made with considerable accuracy.

On the tension tests, a means was sought to determine the modulus of elasticity of the rope as a whole, in addition to the stresses in individual outer wires. One of the most satisfactory methods in use in the past has been described by G. P. Bloomsliter, M. Am. Soc. C. E. (8). He utilized an 8-in. strain-gauge set in holes on brass rings soldered to the rope. Considerable difficulty was encountered due to the untwisting effect of the rope under load, which caused errors in his readings. In order to adapt this method to the present tests, and to minimize such errors, it was decided to use a 10-in. strain-gauge, with holes located on ½-in. square brass lugs, curved to fit the

rope and soldered to it. In this manner, the tendency of the rig to tear away as the rope shrinks under load was eliminated. To compensate for twist, two scales reading to hundredths of an inch were placed 10 in. apart on the rope, and read with the vertical hair of a surveyor's transit; the proper corrections to the measured gauge lengths were then computed after the completion of the test. The stress-strain curve of the rope could then be drawn readily and the modulus of elasticity obtained in the usual manner.

The tests reported by Professor Boomsliter showed quite definitely that the modulus of elasticity of a wire rope, especially one with a hemp center, is a decidedly variable quantity, and tends to increase as the number of loadings increases and as the stress to which the rope is loaded each time is raised. In view of these results, it was decided to investigate more fully this property of wire ropes with hemp centers and to load each tension specimen seven times to approximately 50% of its ultimate load, taking readings on the first, third, fifth, and seventh loadings. As the tests proceeded, it was found advisable to observe also the second loading. All bending specimens were similarly loaded seven times before finally fracturing them, both to insure conditions similar to those in their companion tension specimens and to work the individual wires so as to equalize the stress in them, as indicated by the tensometer readings.

Program of Tests

Because of the large number of variables involved in an investigation of this nature, it was decided to restrict as many of them as possible, and to confine the study to a determination of basic relationships. For this reason, the size of the rope to be tested was set at 1 in., since this was the minimum on which the outer wires extended along the surface far enough to admit attaching a tensometer on a $\frac{1}{2}$-in. gauge length. Similarly, a rope with a hemp center was selected as being more typical than one with an independent wire rope center, and less likely to be confusing in any analysis.

The variables to be investigated were: (1) Sheave diameter; (2) construction; (3) lay; (4) preforming; and (5) grade of steel.

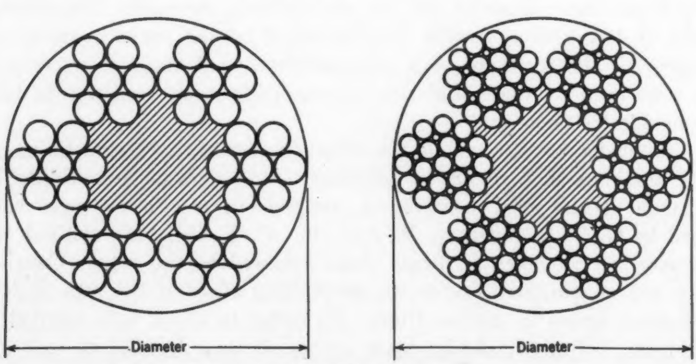


Fig. 1.—6 × 7 and 6 × 19 Wire-Rope Construction.

(*1*) *Sheave Diameter.*—Four values of D were selected: 18 in., 14 in., 10 in., and 7 in., each, measured at the root of the groove.

(*2*) *Construction.*—Ropes of both 6×7 and 6×19 construction were tested (see Fig. 1). The 6×19 ropes contained six filler wires of the same grade of steel as the main wires, added to give a smoother surface to the strand.

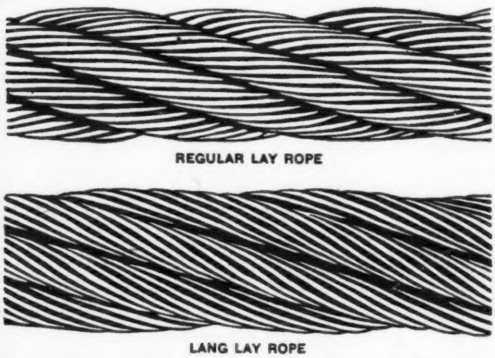


FIG. 2.—REGULAR LAY AND LANG LAY WIRE ROPE.

(*3*) *Lay.*—Both regular lay and Lang lay ropes were included (see Fig. 2). In regular lay ropes the angle of lay of the wire in the strand is equal and opposite to that of the strand in the rope, with the result that the outer wires lie parallel to the axis of the rope. In Lang lay construction, both wires and strands are twisted in the same direction. As the angle of lay was in all cases very close to $18\frac{1}{2}°$, the outer wires in a Lang lay rope were inclined at 37° to the axis of the rope.

(*4*) *Preforming.*—The process of manufacture by which both wires and strands are given an initial helical curvature as they are formed, is known as preforming. This process will be described in detail in succeeding paragraphs. Both preformed and non-preformed types were tested.

(*5*) *Grade of Steel.*—Most of the specimens tested were of cast steel, of the grade produced by almost all wire rope manufacturers, with a specified ultimate strength of 205 000 to 220 000 lb per sq in. A few tests were made for correlation on specimens of plow steel, with an ultimate strength of 235 000 to 250 000 lb per sq in.

Specimens.—Five specimens constituted a set. Of these, one was a tension specimen, 4 ft 6 in. long, and four were bending specimens, 7 ft long, for the four sheave sizes. All the ropes were socketed by means of molten zinc in forged steel open sockets, and all the foregoing dimensions were taken from inside to inside of sockets. These sets were numbered as shown in Table 2.

TABLE 2.—PROPERTIES OF TEST SPECIMENS (HEMP CENTER; d_r = 1 INCH)

No. of set	Type	Lay	Forming
(*a*) 6×7 CONSTRUCTION			
1	Cast steel	Regular	Non-preformed
2	Cast steel	Regular	Preformed
3	Cast steel	Lang	Non-preformed
4	Cast steel	Lang	Preformed
(*b*) 6×19 CONSTRUCTION (SIX FILLER WIRES)			
9	Cast steel	Regular	Non-preformed
10	Cast steel	Regular	Preformed
11	Cast steel	Lang	Non-preformed
12	Cast steel	Lang	Preformed
13	Plow steel	Regular	Non-preformed

For determining the physical properties of the wires which made up these ropes, tensile tests were made on ten samples of each size of wire entering each construction, both of cast and plow steel. These sizes were as shown in Table 3.

TABLE 3.—SIZES OF WIRES

Construction	Number of wires in one round	Diameter, *d*, in inches
6×7	One core wire	0.115
	Six outer wires	0.105
6×19	One core wire	0.073
	Six intermediate wires	0.068
	Six filler wires	0.028
	Twelve outer wires	0.065

Observations were taken on these single wire specimens, of proportional limit, ultimate strength, modulus of elasticity, and location of fracture, and an average stress-strain curve was plotted for each size up to about 85% of the ultimate strength. Average values of these observations were used for determining the physical constants of the wire, and the probable error from this mean was noted.

MANUFACTURING PROCESSES

The various manufacturing processes and machines used in the production of wire and wire rope have been rather fully described elsewhere, notably by Messrs. Carstarphen (1) and Meals (1). The process is reduced essentially to three steps: First, the drawing and treating of the wire; second, the spinning of these wires into a strand of the desired size and construction; and, third, the closing of several strands around a hemp or wire rope center to form a wire rope. It is during this last step that the ropes are preformed, if so desired, so that the wires and strands are permanently deformed and lie in the finished rope with no tendency to unravel or kink.

Fig. 3 shows a large vertical closing machine at the point where six strands are drawn through a die over a lubricated hemp center to form a non-preformed, regular lay rope. The frame and die are held stationary, and the slotted cone and spools from which the strands are drawn rotate. In addition, the spools are given a planetary motion so that the strands are laid into the rope without any twist, and their lay is controlled by the speed with which the finished rope is withdrawn. For forming Lang lay rope, the frame is rotated in the reverse direction, and the spools given a back turn to minimize the untwisting effect.

The contrast between this method and that used in making preformed ropes is illustrated in Fig. 4. The frame and die are the same, but, in this case, the smooth cone is replaced by one on which are mounted three small sheaves for each strand. These sheaves are placed as shown in Fig. 4 and the strands threaded around them so that a helical permanent set is imparted to them. This set is noticeable in the section of the strands just as they enter the closing die. The proper position of the small sheaves must be determined very exactly in order that the helix will be of the exact size

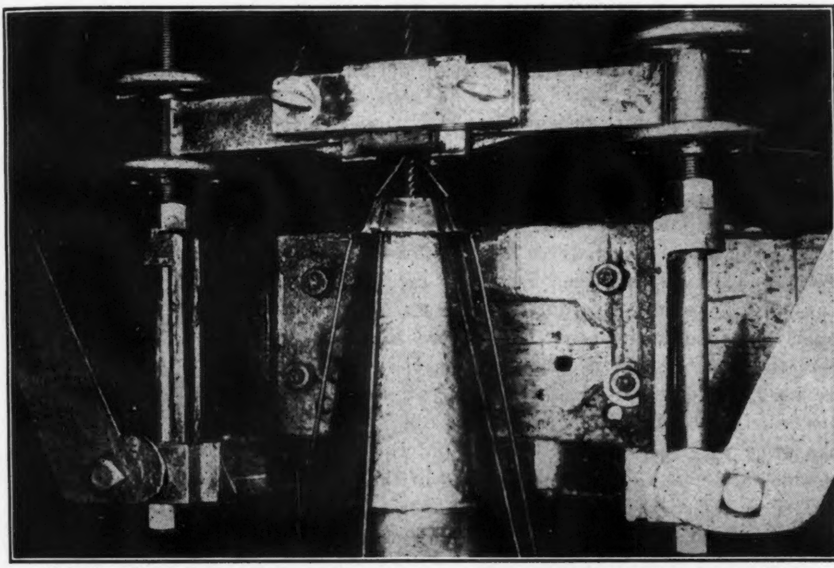


FIG. 3.—WIRE ROPE MACHINE FABRICATING A REGULAR LAY, NON-PREFORMED ROPE.

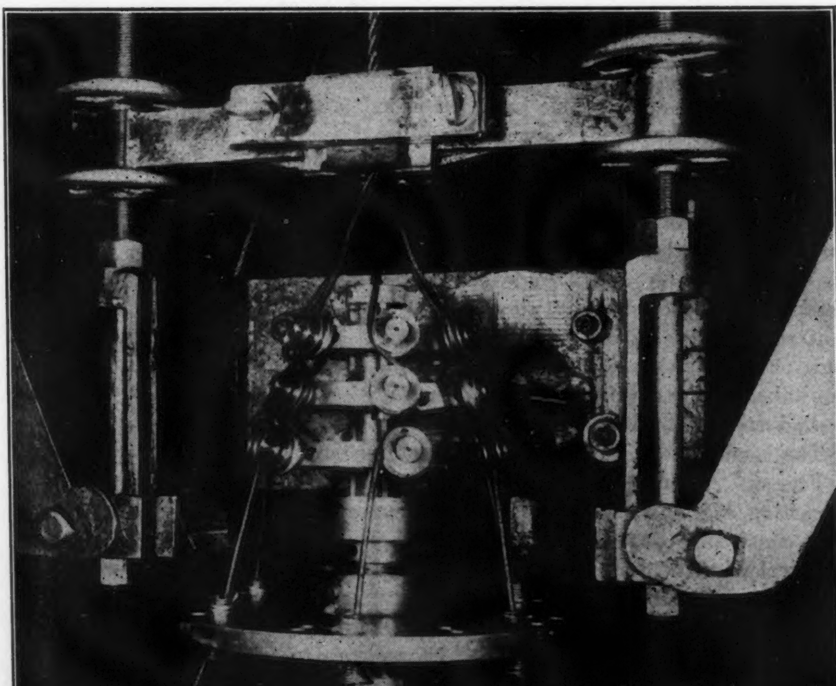


FIG. 4.—WIRE ROPE MACHINE MODIFIED FOR FABRICATING A REGULAR LAY, PREFORMED ROPE.

required for forming the desired rope. In preforming Lang lay ropes, these small sheaves are replaced by spiral holes through which the strands are drawn, in order to minimize the twisting action to which this type is subject.

Testing Apparatus

All specimens both in tension and in bending were tested in a 300 000-lb testing machine, which was calibrated to 200 000 lb and found to be correct within 0.25 per cent. For the tension tests, the sockets were passed through the holes in the two heads of the machine and secured by steel plates, in which $1\frac{3}{4}$ - in. holes had been drilled to receive the socket pins. Brass gauge points for the gauge were cut from a section of 1-in. brass pipe, and when properly cleaned with emery paper were soldered to the rope. In all cases except the first test, where six gauge lengths were provided, two gauge lengths were used, directly opposite each other at about the center of the specimen. By exercising proper care in soldering, only one of all the brass gauge points broke away in the course of testing, and the fracture of the rope was never traceable to the heat treatment of wires in the vicinity of the soldering operations. For measuring the twist, two paper scales graduated to 0.02 in., were affixed to one side of the cable, just under each gauge point, by rubber bands, and these scales were read to 0.01 in. on the vertical hair of a surveyor's transit set up on a near-by table. The gauge holes in the brass plugs were also aligned vertically with this transit when drilled.

The operation of attaching the tensometers proved to be the most difficult part of the tension set-up, as the ropes contracted appreciably under load, causing the gauges to become loose. However, after some experimentation, it was found that for regular lay ropes a pair of tensometers could be mounted opposite each other on a standard gauge-holder, and could be held in place by connecting the far ends of the holder by a short strong spring. This arrangement gave consistently good readings even at very high loads. For Lang lay ropes this device could not be used, however, due to the fact that the outer wires lay at an angle of 37° to the axis of the cable. For this reason the two tensometers were mounted separately, on fittings especially built to hold the gauges at the required angle and allow some play as this angle changed under load. On many of the tension specimens measurements were taken of the diameter before loading and at nearly full load, with a pair of slide calipers, to determine the decrease in diameter under load.

The bending tests required the construction of a special testing rig shown in Fig. 5. The upper head of the testing machine was removed from its supporting columns, and across two of them was placed diagonally a framework consisting of two 15-in. channels, held vertically in place by $\frac{1}{2}$ - in. plates welded to their ends and separated by a slot $2\frac{1}{2}$ in. wide. At the middle of the top face of these channels were welded two semi-circular bearing blocks, cut to fit the 4-in. steel pin which served as an axle for the sheaves. This allowed the sheaves to pass through the slot between the channels, leaving the top half, over which the cable passed, free for the mounting of gauges. The lower ends of the rope passed downward through the slot and the sockets were held

by pins in plates welded to a short section of reinforced H-beam. To the lower face of this beam was welded a vertical steel piece which, in turn, was gripped by the jaws of the testing machine. The sheave at all times was free to rotate on its pin and the pin in its bearing-blocks; and from the gauge readings it is believed that very nearly the same stress was developed in the rope on each side of the sheave at all times.

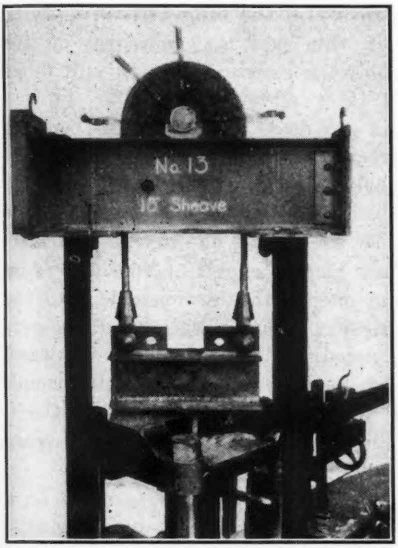


FIG. 5.—TESTING RIG FOR WIRE ROPE SPECIMEN OVER 18-INCH SHEAVE.

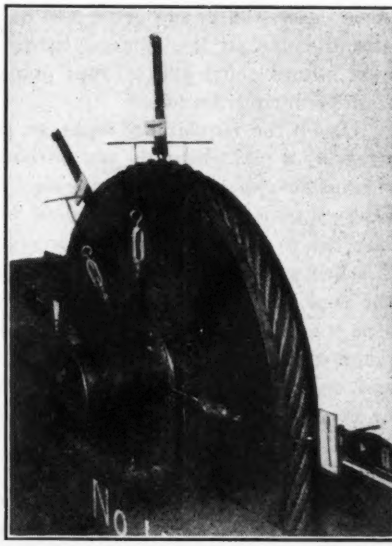


FIG. 6.—CLOSE-UP VIEW OF BENDING SPECIMEN, SHOWING MEANS OF ATTACHING TENSOMETERS.

The four sheaves were machined from solid steel plates, 2 in. in thickness, and in accordance with modern practice, as noted by Mr. Meals (7), the grooves were made $1\frac{1}{16}$ in. in diameter, and semi-circular to facilitate measurements on the ropes. A 4-in. hole was cut in the center of each sheave, and carefully machined so that the steel axle could be inserted easily by hand which insured a snug fit.

It was decided to place the four tensometers available, one at the top of the sheave, one at the 45° point, and one at each tangent point, since a good average reading was required at the latter point because of the high stresses present. Several ideas were tried for holding the gauges in place and, at the same time, meeting the problems of shrinkage and sliding along the sheave due to tension. As finally arranged (see Fig. 6), the apparatus consisted of two slotted steel rings which were slipped over the axle on either side of the sheave. To these were attached short, stiff springs, and these, in turn, carried small turnbuckles and wire loops of varying length. Small rods were passed through the holes in the gauges, in the case of regular lay ropes, and slipped through these wire loops (Fig. 6). The gauges were held firmly in place by tightening the turnbuckles until there was an appreciable tension

in the springs on either side, and the gauge was free to move slightly along the sheave as the rope stretched. Various combinations of length of springs and wire loops enabled this apparatus to be used on all four sheave sizes.

For the Lang lay ropes the rig used was identical except that special holders of welded construction had to be devised for keeping the gauges fixed at an angle of 37 degrees. Although it was a rather delicate matter to set up the gauges for a test, this apparatus gave consistently good results. In some cases, the gauge could not be set directly at the tangent points, as no strand came to the surface there, and in this case they were set on the next strand above the tangent point, the stretch causing them to pull down slightly during the test.

On all the regular lay ropes, an attempt was made to evaluate the bending stress by a plain bending test without tension. For this purpose an auxiliary rig was devised, consisting of a steel plate bolted fast in a horizontal position to a heavy table. In this plate were drilled holes into which steel pins could be inserted to simulate sheave diameters of 50, 25, 18, 14, 12, 10, $8\frac{1}{4}$, and 7 in. A space was left clear in the center to allow placing a pair of tensometers on the rope, one on the compression side and one on the tension side, and the rope was bent over these diameters in succession by hand, while readings were taken of the strains that occurred. This apparatus was inherently awkward, and only by averaging a great number of results could any definite trends be established. The rig was not adapted for Lang lay ropes, as the attachments necessary for holding the gauges in place would not fit in the space allowed.

Single wire tests for determining the stress-strain curve were made on a 2 000-lb, hand-power testing machine, with specimens about 15 in. long.

Discussion of Test Data

The great mass of data taken during these tests does not permit the inclusion of all the test results. Accordingly, an attempt has been made to follow through the procedure in one particular typical instance, giving all the results and curves obtained, and to summarize the results in the form of curves for the remaining sets of ropes. Particular exceptions or variations from the typical results are noted and in some cases illustrated.

Tension Tests.—A typical set of curves was obtained in these tests on Set No. 2, a 1-in. 6 × 7 cast, regular lay, preformed rope. The strain readings on the rope as a whole, corrected for twist, have been plotted in Fig. 7 for the seventh loading. Similar curves were plotted for each of the preceding loadings in which the load was carried to only 35 000 lb, and the modulus of elasticity noted in each case. The first rope tested (Set No. 13) was arranged with three gauge lengths on each side. The center set showed values of the modulus about 3% greater than those at either end, and in the belief that this center value was more truly representative, only one pair of gauge lines, located at the center of the specimen, was used in all future tests.

The reasons for the initial curvature of the load-strain curve, shown plainly in Fig. 7, are discussed by Messrs. Griffith and Bragg (6). Their con-

clusion is that at low loads the elongation under stress is not wholly elastic, due to the presence of initial curvature in the strands and wires from the laying and a certain "slack" or curvature in the rope itself.

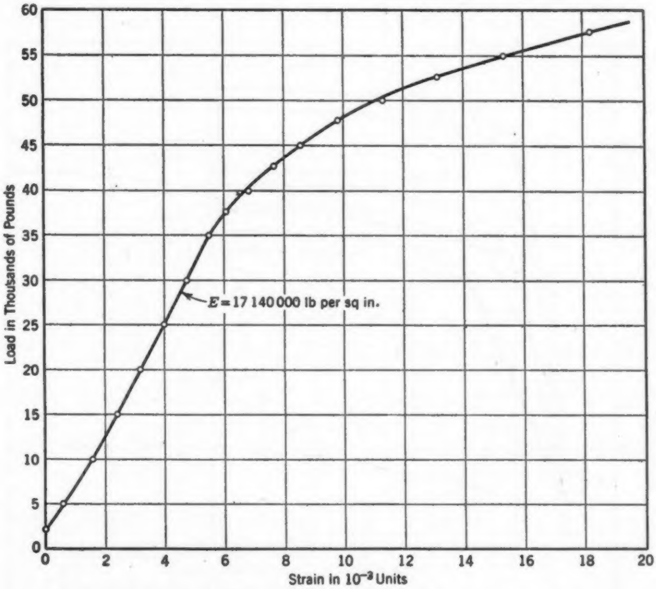


FIG. 7.—SET NO. 2: LOAD STRAIN CURVE OF 1-INCH, 6 × 7, CAST-STEEL REGULAR LAY, PREFORMED WIRE ROPE; SEVENTH LOADING; NET AREA = 0.374 INCH².

Fig. 8 presents curves showing the rise in modulus of elasticity with repeated loadings for all sets of ropes tested (see Table 2). The sharp initial rise in all these curves after the first loading is due to the large initial consolidation of the hemp center, and the compacting of the wires and strands. It will be noted that, in general, the 6 × 7 ropes (Sets Nos. 1 to 4) show values greater than the 6 × 19 construction. Furthermore, the Lang lay ropes (Sets Nos. 3, 4, 11, and 12) show a slightly higher modulus than the regular lay ropes, and the preformed ropes (Sets Nos. 2, 4, 10, and 12) seem to run higher than the non-preformed types. The one plow-steel specimen (Set No. 13) had a slightly higher modulus than its companion cast-steel rope.

The 1-in., 6 × 19 cast, Lang lay, non-preformed rope (Set No. 11) was tested for seven loadings to 10 000 lb (14% of the ultimate load), with a view to determining whether the same increase in modulus occurred at working loads as at relatively high loads. The seventh loading was then continued to 35 000 lb, or 49% of the ultimate, which was repeated until the eleventh loading, when the test was carried to destruction. Fig. 8 shows that at working loads a rise in modulus of elasticity does occur, but that the values are much lower than when the load is increased to about the proportional limit. This is explained by the fact that at these low loads the stress-strain curve was still concave upward. The value selected as the modulus in such cases was

the slope of the tangent to the curve at the maximum load. In every case, the slope was less than that found when the load was further increased, indicating quite definitely that the rope had not yet, at the low loads, reached a period of elastic behavior.

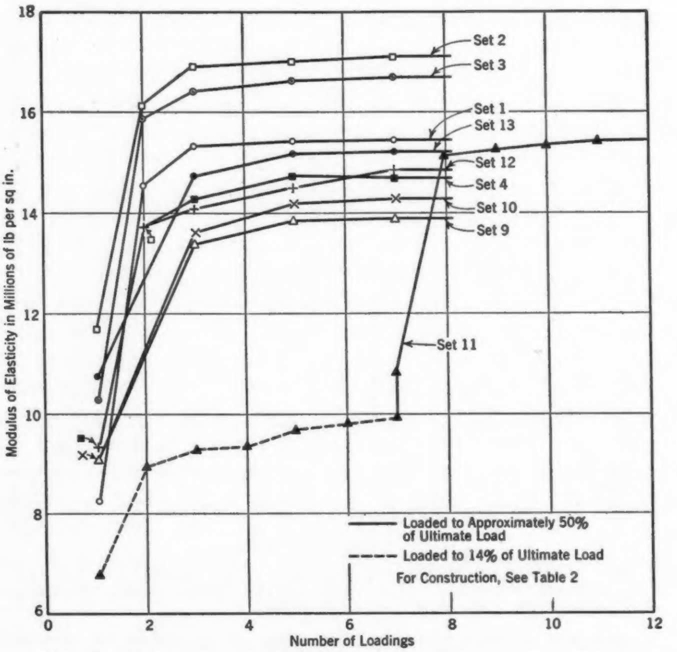


FIG. 8.—VARIATION IN MODULUS OF ELASTICITY OF WIRE ROPE WITH REPEATED LOADINGS.

For the tension test of Set No. 2, Fig. 9 shows the load-strain curves for the average of two individual wires, as shown by the tensometers, for each loading. On the first loading, both gauges ran off the scale early in the test, but succeeding loadings show the wires to be taking stress in a very uniform manner. It is evident that the first loading tends to redistribute and equalize the stress in the several wires and further loadings bring the rope to an almost perfectly elastic state as regards these stresses. The fact that the curve for the seventh loading breaks away at exactly 35 000 lb is indicative that the proportional limit of some of the wires had been passed on preceding loadings, and that these had become slightly strain-hardened and the proportional limit raised accordingly.

The strains for the seventh loading (Fig. 9) were transformed into stresses as explained previously by use of a stress-strain curve for the single wire. Fig. 10 shows the stress-strain relations for this particular size of wire, 0.105 in. in diameter, cast steel.

The results of this transformation are expressed in the form of a load-stress curve, as shown in Fig. 11(*a*). It will be noted that up to the proportional

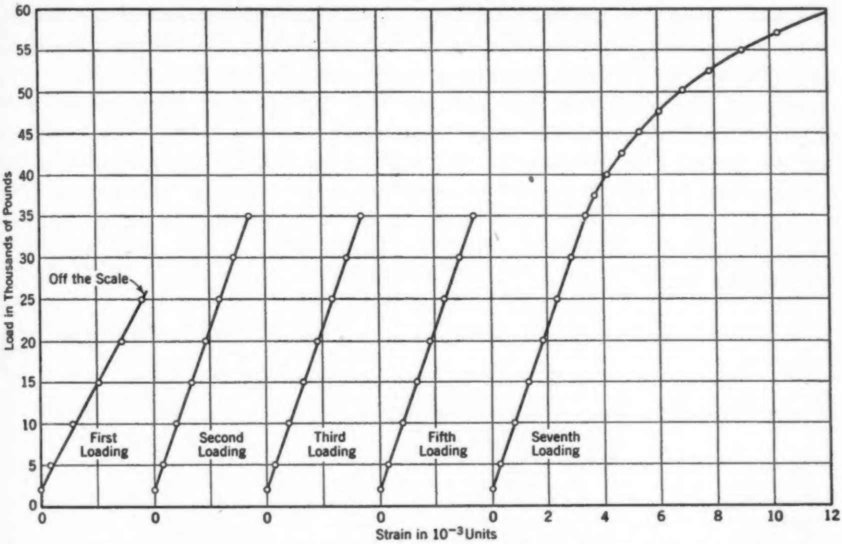


FIG. 9.—TENSION TEST, SET NO. 2; LOAD-STRAIN CURVES OF A SINGLE WIRE; 6 × 7 CAST-STEEL, REGULAR LAY PREFORMED WIRE ROPE.

limit the curve follows very closely the dotted line representing the load divided by the net area of steel, which is the curve for a homogeneous bar of the same area of cross-section. Beyond this point there is a reverse curve (although this is not present in every case). When extrapolated to a value of the load equal to the observed ultimate load on the rope, this reverse curve shows a stress value of 219 500 lb per sq in., which checks very closely the average single-wire strength for this grade, namely, 219 000 lb per sq in. As always, in extrapolating curves, there is a chance for error, but as every curve showed this ultimate stress to be close to 219 000 lb per sq in., these load-stress curves seem to be well established.

The curve in Fig. 11(*a*) is typical of all those obtained on regular lay ropes. On Lang lay ropes, a different kind of curve was obtained, as is illustrated by Fig. 11(*b*), for 1-in., 6 × 19

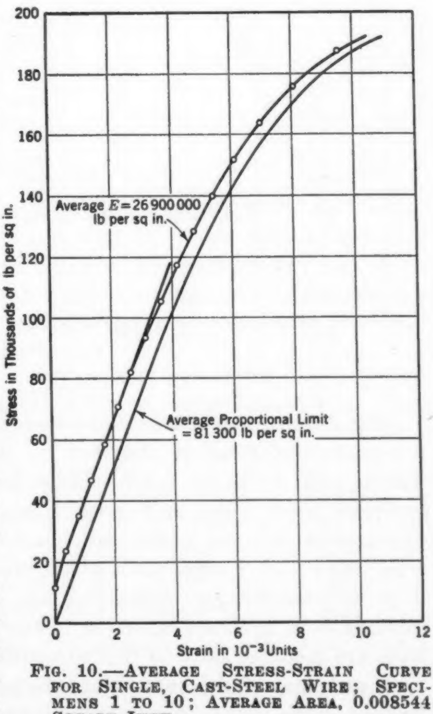


FIG. 10.—AVERAGE STRESS-STRAIN CURVE FOR SINGLE, CAST-STEEL WIRE; SPECIMENS 1 TO 10; AVERAGE AREA, 0.008544 SQUARE INCH.

cast-steel, Lang lay, preformed rope. In this case the stress in the outer wires is considerably lessened, and falls well below the dotted line for load divided by net area for the greater part of the test, but picks up rapidly at the end. In the case of Fig. 11(*b*), the stress below the proportional limit is 0.803 times that given by the dotted line, which may be assumed to be that for the regular lay rope. Theoretically, this factor should be cos 37°, or 0.799, since the outer wires are inclined at 37° to the axis of the rope. The average value of this ratio for all the Lang lay ropes tested was 0.783, and this agreement is within the limits of experimental error.

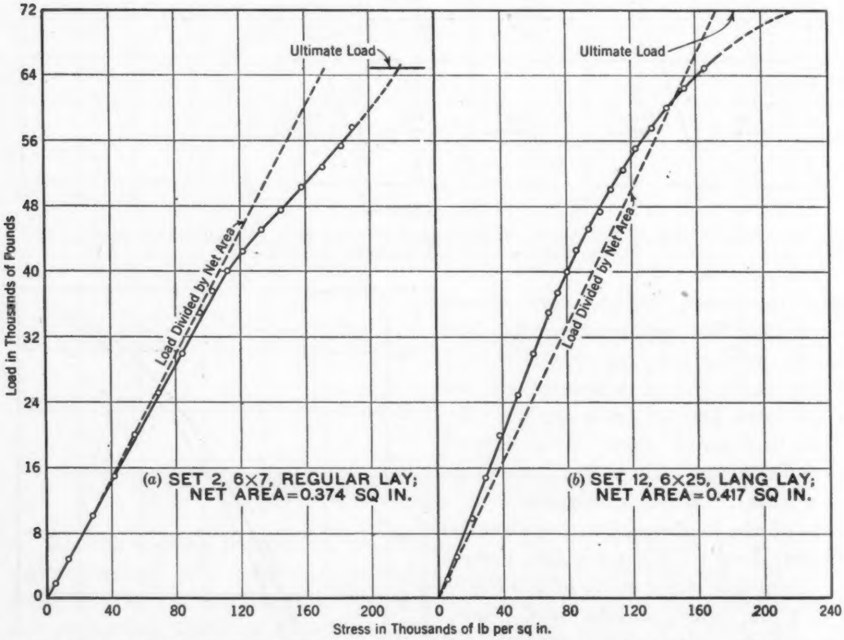


FIG. 11.—LOAD STRESS FOR TENSION TEST; 1-INCH, CAST-STEEL WIRE ROPE, PREFORMED.

The difference in load-stress curves for the tension tests of preformed and non-preformed ropes is obscured by the incidental variations of each test. There seems to be no appreciable difference in the stress conditions for the two cases when loaded up to seven times, although some tendency for a quicker equalization of stress in the individual wires has been noted for the preformed type. The 6 × 7 ropes show a less steep load-stress curve than those of the 6 × 19 construction, indicating the presence of higher stresses, but this increase is in inverse proportion to the net area of section, and the foregoing facts are modified only in this proportion.

A summary of the untwisting effect observed for every rope is presented in Table 4. The values given are in inches of circumferential twist in a 10-in.

gauge length, and the load to which each loading was taken is also recorded. These values show the decrease in twist with repeated loadings and are useful for purposes of comparison. The preformed ropes (Sets Nos. 2, 4, and 10) in general show less twist than the non-preformed ropes, with the exception of Set No. 10, which is about the same as Set No. 9. The Lang lay ropes (Sets Nos. 3, 4, 11, and 12) do not show any greater untwisting than regular lay ropes; in fact, Set No. 12 shows much less twist than its companion specimen, Set. No. 10. It must be remembered, however, that the ends of these specimens were totally fixed against rotation under load, by the frictional forces acting on the heads of the testing machine. Ropes of 6 × 7 construction (Sets Nos. 1, 2, 3, and 4) seem to untwist about as much as those of the 6 × 19 construction.

TABLE 4.—Inches of Circumferential Twist in 10-Inch Gauge Length

Set No.	Load, in thousands of pounds	Number of Loading												
		First	Second	Third	Fourth	Fifth	Sixth	Seventh		Eighth	Ninth	Tenth	Eleventh	
								Final load, in thousands of pounds	Twist, in inches				Final load, in thousands of pounds	Twist, in inches
1	35	0.11	0.06	0.05	....	0.05	....	60	0.10	....	....	....	....	....
2	35	0.04	0.01	0.01	....	0.02	....	57.5	0.02	....	....	....	....	....
3	35	0.09	0.07	0.06	....	0.03	....	60	0.09	....	....	....	....	....
4	35	0.03	0.02	0.02	....	0.01	....	57.5	0.08	....	....	....	....	....
9	35	0.08	....	0.05	....	0.06	....	65	0.12	....	....	....	....	....
10	40	0.15	....	0.07	....	0.08	....	60	0.13	....	....	....	....	....
11	10	0.03	0.02	0.02	0.02	0.02	0.02	....	0.03	....	....	....	....	....
11	35	....	....	....	....	....	....	....	0.09	0.09	0.07	0.08	65	0.13
12	35	0.08	0.04	0.05	....	0.02	....	65	0.06	....	....	....	....	....
13	50	0.14	....	0.08	....	0.07	....	64	0.10	....	....	....	....	....

Data on the shrinkage of wire ropes due to consolidation of the hemp center are available for most of the specimens tested. These data are assembled in Table 5, the diameters being recorded to the nearest 0.01 in., and show that a rope will acquire a permanent decrease in diameter of 1% to 2% when loaded to 50% of its ultimate load, and that the total decrease at fracture is about 5 to 6% of the original diameter.

TABLE 5.—Decrease in Diameter (Inches) of Ropes Under Load

Set No.	Average original	At beginning of final loading	During Final Loading	
			Load, in pounds	Diameter, in inches
1	1.01	....	60 000	0.96
2	1.02	....	57 500	0.97
3	1.00	0.98	62 500	0.95
4	1.00	0.98	60 000	0.95
9	1.02	....	68 000	0.99
11	1.02	1.01	65 000	0.98
12	1.04	1.02	65 000	0.99

The efficiency of a wire rope in tension is its ultimate load divided by the product of the net area by the ultimate strength of the wire, the denominator

being the theoretical maximum load that a homogeneous rod could attain. Values of the efficiencies obtained in these tests are tabulated in the "Summary".

Bending Tests.—Set No. 1 (Table 2), has been selected as typical of the bending test results obtained. Observations were made of the strains at the top of the sheave for the first, third, and fifth loadings, and of the strains at the top, 45° point, and the two tangent points for the seventh loading to destruction. The load-strain curves for these loadings of the specimen bent over the 18-in. sheave, are shown in Fig. 12(*a*); very similar curves were obtained over the other three sheave sizes. The friction effect on stress at the top of the sheave is well illustrated by the curves for the first, third, and fifth loadings. As all the stresses are below the proportional limit, these are also load-stress curves to another scale. Upon release of the load, the stress at the top point remained constant until the friction load caused by stretching the cable decreased to zero, and built up until slipping occurred along the sheave in the reverse direction. Thus, this drop is a measure of twice the frictional force present. According to the foregoing reasoning, the values of strain at the tangent points should follow back the original curve as the load is removed, with no hysteresis, and this has been found to be the case. On practically every one of the Lang lay ropes, an initial compression was observed on the first loading, which is evidently due to poor initial stress distribution inherent in this type of construction.

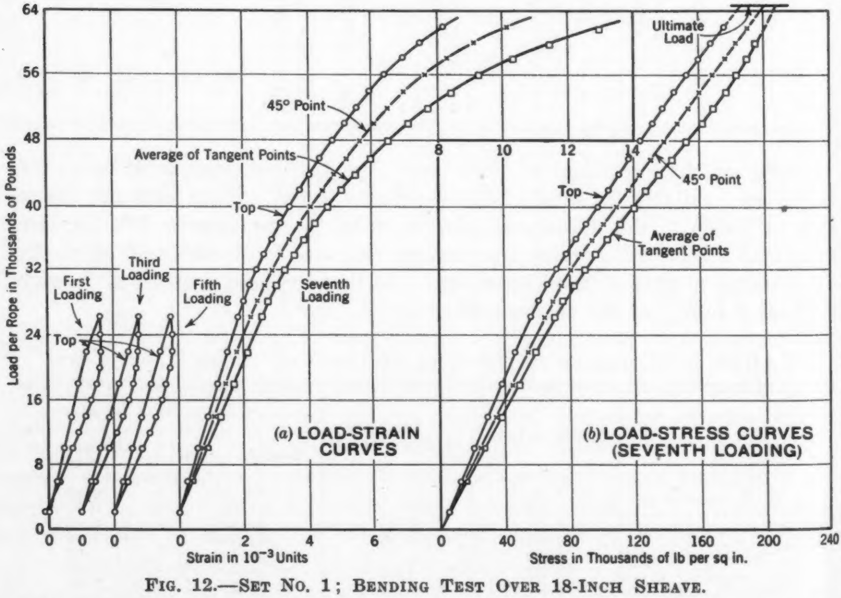


FIG. 12.—SET NO. 1; BENDING TEST OVER 18-INCH SHEAVE.

The transfer of strain values to stresses as described for the tension test was made for the seventh loading of Fig. 12(*a*), giving the set of load-stress curves shown in Fig. 12(*b*). These curves show very definitely that the maxi-

mum stress lies at the tangent point, where the rope meets the sheave, and that the stress decreases progressively up and around the sheave, due to the increasing frictional forces present, to a minimum value at the top. The coefficient of friction of rope on the sheave was determined in each instance by the formula[4]:

$$\frac{s_g}{s_p} = e^{0.5\pi f} \text{..........................(13)}$$

Two values of the stress ratio for each sheave size were taken from the load-stress curves, one at the proportional limit and one near the ultimate load; in most cases these values were in reasonable agreement. The values of f for all four sheave sizes of each set were then averaged, with the results shown in

TABLE 6.—COEFFICIENT OF FRICTION

Regular Lay Ropes		Lang Lay Ropes	
Set No. (see Table 2):	Coefficient, f	Set No. (see Table 2):	Coefficient, f
1	0.146	3	0.432
2	0.156	4	0.344
9	0.101	11	0.392
10	0.149	12	0.348
13	0.133	.....	.....
Average	0.137	Average	0.379

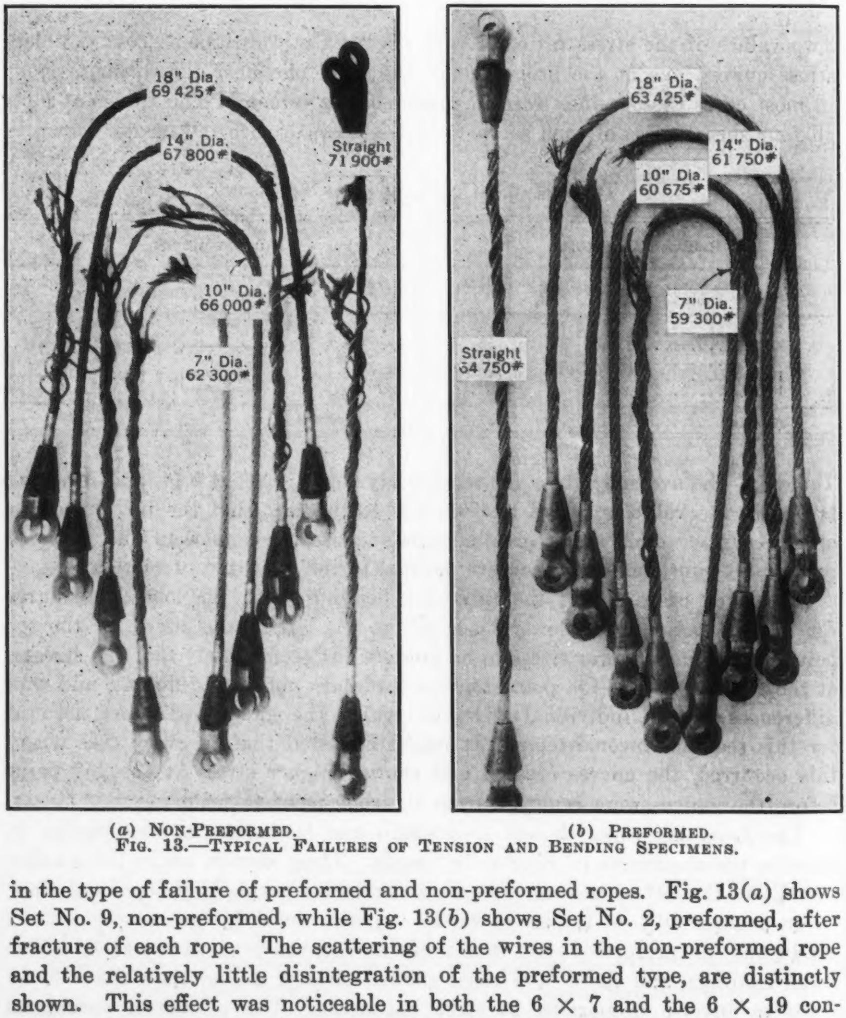
Table 6. The average values for regular lay ropes is about 0.14, and for Lang lay ropes the value is raised to 0.38, due to the fact that the inclined wires on the surface offer much greater resistance to slippage than the wires of regular lay construction, which are parallel to the direction of motion.

In eleven cases out of the thirty-six bending tests, the load-stress curve for the 45° point at the lower loads fell to the left of the curve for the top point, indicating a lower stress to be present. It seems likely that the stresses at the top and at the 45° point for low loads are not very different, and that differences in the individual wires on which the gauges were set account for this seeming inconsistency. It might be noted that in every case where this occurred, the curves crossed and showed higher stress at the 45° point before the gauges were removed from the rope prior to failure.

The Lang lay ropes showed load-strain and load-stress curves similar in form to the specimens of regular lay ropes. These curves, as in the tension test, lie to the left of the corresponding curves for regular lay rope, and show on an average 80% of the stress values. Again, the difference between stress conditions in preformed and non-preformed ropes was not marked after seven loadings, and the 6 × 7 ropes showed stresses higher than the 6 × 19 ropes in inverse proportion to their net areas. The plow-steel specimens showed exactly similar effects, with loads and stresses raised in proportion to the ultimate strengths of the single wires.

[4] "Applied Mechanics", by A. P. Poorman, M. Am. Soc. C. E., Second Edition (1932), p. 132.

The manner in which the bending specimens failed supports the observations drawn from the preceding curves that the greatest stress occurs at the tangent points. With a very few exceptions, all bending specimens failed at one of the tangent points. The only exceptions were those that failed at the socket, which failure occurred for only two specimens of Set No. 12. This fact is demonstrated clearly in Fig. 13, which also illustrates the difference



(*a*) Non-Preformed. (*b*) Preformed.
Fig. 13.—Typical Failures of Tension and Bending Specimens.

in the type of failure of preformed and non-preformed ropes. Fig. 13(*a*) shows Set No. 9, non-preformed, while Fig. 13(*b*) shows Set No. 2, preformed, after fracture of each rope. The scattering of the wires in the non-preformed rope and the relatively little disintegration of the preformed type, are distinctly shown. This effect was noticeable in both the 6×7 and the 6×19 constructions, although to a greater extent in the latter. The decrease in strength as the sheave size decreases, is demonstrated by the ultimate loads in Fig. 13. All the ropes of 6×19 construction failed gradually, snapping wires being heard in the interior of the specimen at loads considerably below the ultimate,

in some cases as much as 10 per cent. The 6 × 7 ropes, however, contained only one core wire to a strand, of quite large diameter, and these ropes failed suddenly with little or no warning. In general, two to four strands were broken, and in only one instance were all six strands of the rope broken simultaneously.

The striking similarity between the curves for stress at the tangent point over all four sheave sizes and the corresponding curve for the tension test, led to the plotting of these values for Set No. 1 on the same coordinates, as shown in Fig. 14. All the curves are seen to coincide within the range of experimental errors, and this was found to be the case for all the ropes tested, although the agreement was not so perfect for the Lang lay ropes. This demonstrates that the bending stress is not increased after the rope is initially bent over the sheave, but that thereafter the rope behaves exactly as in a tension test. Bending over a given sheave, therefore, is equivalent to shifting the curve of Fig. 14 to the right by a constant amount, thus causing the curves to intersect the vertical line representing their ultimate strength at successively lower values of the load as the bending stress increases. This fact served as a basis for a graphical determination of the bending stress, by extrapolating the load-stress curve to the breaking load and subtracting the stress there observed from the known ultimate strength of 219 000 lb per sq in. Although this method is admittedly crude, its accuracy in determining bending stress may be judged from the values recorded in the table on correlation of bending stress formulas, given in the "Summary".

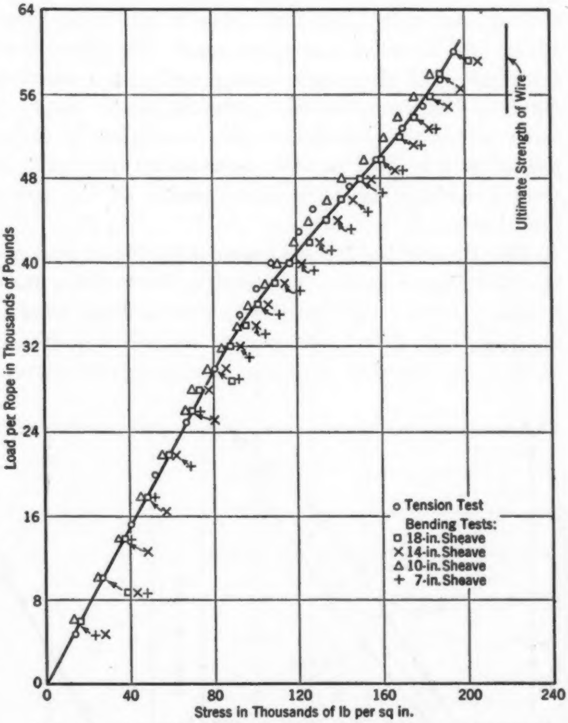


FIG. 14.—COMPOSITE LOAD STRESS CURVE FOR TENSION AND BENDING TESTS, SET NO. 1.

The shape of the load-stress curve in all cases is essentially the same. One would normally expect this to be a straight line for a homogeneous material, but in a composite body, such as a wire rope, opportunity is given

for some wires to yield more than others and thus to redistribute the stresses in a strand. An explanation of the observed fact that beyond the proportional limit the outer wires take more than their proportional share of the stress, as shown by the breaking away of the curves to the right of a straight line, is found in a theoretical analysis of stress distribution in a strand, presented by Messrs. Griffith and Bragg (6). They showed that the stress in a wire of a given ring is directly proportional to $\cos^2 a_n$, in which a_n is the angle of lay of the wires in the strand. The ratio of stress in the outer wire to stress in the core wire, therefore, should be 0.899, below the proportional limit. When the inner wire or wires reach this point, they begin to yield first, and this ratio will rise, approaching unity as a maximum, until theoretically at fracture the stress in all wires should be equal. Actually, certain weaker inner wires will fail before this condition is realized. This theory is supported by the experimental observations that on 6 × 19 ropes snapping wires were invariably heard in the interior of the specimen at loads well below the ultimate.

For illustration an exaggerated condition was chosen, in which the strain in the outer wire was assumed as 0.74 times the strain in the core wire. Proper values of the stress were then selected from a typical wire stress-strain diagram, and these were plotted against percentage of total load, as shown in Fig. 15, together with the average stress curve, which is a straight line,

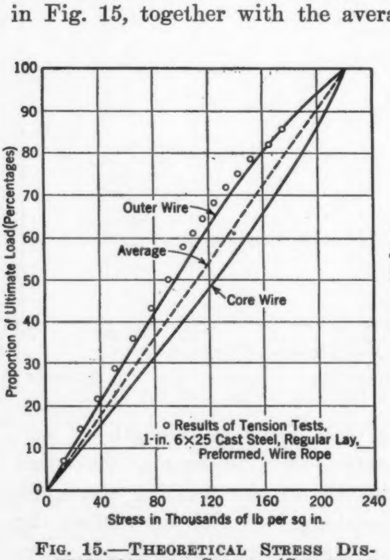


FIG. 15.—THEORETICAL STRESS DISTRIBUTION IN A STRAND (STRAIN IN OUTER WIRE = 0.74 STRAIN IN CORE WIRE.)

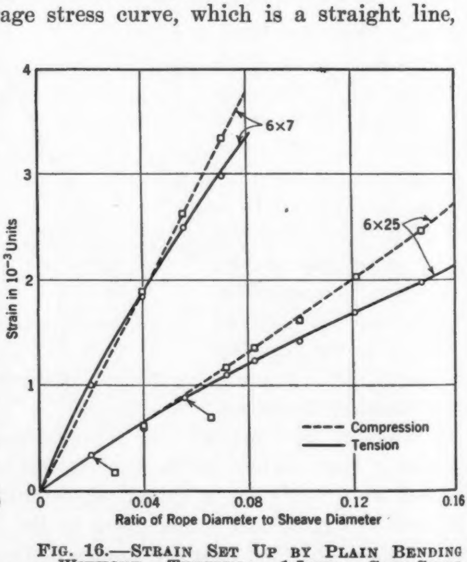


FIG. 16.—STRAIN SET UP BY PLAIN BENDING WITHOUT TENSION; 1-INCH CAST-STEEL REGULAR, NON-PREFORMED WIRE ROPE.

as would be expected. The similarity of the curve marked "Outer Wire" to the observed load-stress curves is notable, and for comparison there has been included on this plot the results of the tension test of the 1-in., 6 × 19 cast-steel, regular lay, preformed specimen. It is evident that the assumption

that the outer wire takes 74% of the stress in the core wire, below the proportional limit, was none too extreme.

The results of the plain bending test without tension, in which the regular lay ropes were bent by hand over steel pins, are represented typically by Fig. 16, for 1-in., cast-steel, non-preformed ropes of 6 × 7 and 6 × 19 construction. Each point is the average of two readings on each of four ropes, a total of eight readings. These curves are interesting in that they show that the stress on both the tension and the compression sides of the rope increases nearly linearly with the ratio of rope diameter to sheave diameter at the root. Although in every case the compressive stress exceeds the tensile stress, this is of little importance, as the addition of direct stress in the form of a pull will decrease this stress and soon bring these wires into tension as well. When these strains are transformed into stresses, another value is obtained for the bending stress, and these results have also been listed in tabular form in the "Summary". The values for the 6 × 7 ropes were much higher than those for the 6 × 19 construction, and the method broke down completely for these values. However, the trend is notable, and, as will be shown subsequently, the loss of strength of a rope varies in exactly this manner with the ratio of rope diameter to sheave diameter.

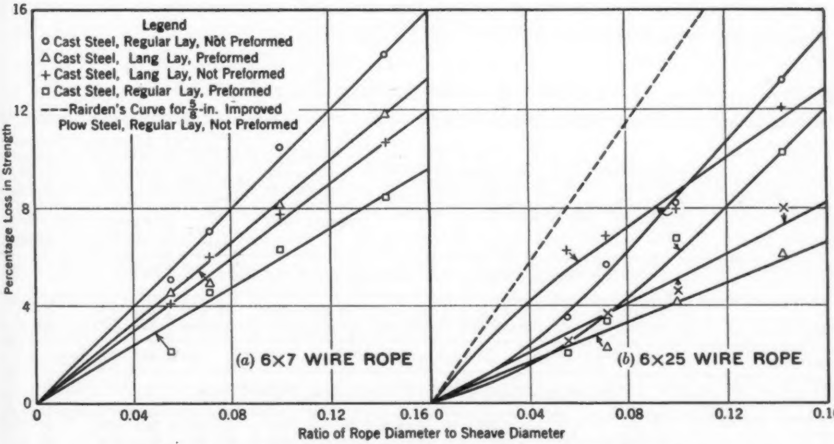


FIG. 17.—LOSS IN STRENGTH OF WIRE ROPES IN BENDING.

A summary is presented in Fig. 17 of the loss in strength in bending plotted against the ratio of rope diameter to sheave diameter for every set of ropes tested. A curve presented by Mr. A. S. Rairden (1) for tests on $\frac{5}{8}$-in.. 6 × 19 plow-steel, regular lay, non-preformed ropes, is also included with the reciprocal of his ordinate scale used in this case. It is believed that by inverting this ratio (thus converting a hyperbolic curve into a straight line), the curve may be fitted much more easily to a number of fairly erratic points, as the origin is fixed and only one degree of freedom is allowed in locating the curve. The curved relationship shown for some of the sets may very well be due to an inaccurate value for the tension test, which affects all the points,

and is equivalent to shifting the curve up or down. Such a shift has been made in the case of Set No. 11, where the tension specimen failed at the socket at a load of 71 650 lb, without developing its full strength. This load was so low as to indicate an actual increase in strength over the 18-in. and the 14-in. sheaves, and in view of the relationships amply demonstrated in the other ropes, the value has been raised to 75 000 lb by shifting the straight line to pass through the origin, and the loss in strength computed on this basis. The short arrows on this plot serve merely to identify each point with its proper curve, and do not indicate that the point itself has been moved.

It may readily be seen that the non-preformed ropes show a greater loss in strength in every case than the preformed types. Similarly, in all but one instance, the Lang lay ropes show less loss in strength than their corresponding specimens of regular lay, due to the lower bending stresses present. There seems to be very little difference between the results for the 6 × 7 and 6 × 19 constructions and for the cast and plow grades of steel, when the loss is considered on a percentage basis. Considerably more data are needed, however, before any definite conclusions can be reached on these points.

A third method used in determining the bending stress in the rope is to divide the loss in strength by the product of net area and efficiency in tension, the denominator being the effective net area resisting bending. The results are shown in the "Summary".

TABLE 7.—SINGLE-WIRE TEST DATA

Set No	Nominal diameter, in inches	No. of specimens	Average diameter, in inches	Average area, in square inches	Average modulus of elasticity, in pounds per square inch	Average proportional limit, in pounds per square inch	Average ultimate strength, in pounds, per square inch	Proportional limit ÷ ultimate strength (percentages)
(a) Wires Used in 6×7 Cast-Steel Wire Ropes								
3	0.105	10	0.1043	0.008544	26 900 000	81 300	215 400	37.7
3	0.115	10	0.1142	0.010243	28 910 000	53 900	220 200	24.5
Average	....	..	......	....	....	......	217 800	....
(b) Wires Used in 6×19 Cast-Steel Wire Ropes								
3	0.065	10	0.0649	0.003308	26 820 000	91 000	221 600	41.1
3	0.068	10	0.0671	0.003536	25 090 000	82 800	212 600	38.9
3	0.073	10	0.0729	0.004174	26 830 000	83 900	228 800	37.5
Average	.....	..	......	....	....	......	219 300	....
Average, all cast-steel specimens	.....	..	......	....	26 910 000	78 600	218 700	35.9
(c) Wires Used in 6×19 Plow-Steel, Wire Ropes								
5	0.028	10	0.0283	0.000629	....	....	248 000	....
5	0.065	10	0.0641	0.003227	26 050 000	107 000	245 800	43.5
5	0.068	10	0.0678	0.003610	26 840 000	88 700	250 600	35.4
5	0.073	10	0.0734	0.004231	26 390 000	87 000	241 700	36.2
Average	.....	..	......	....	....	....	246 500	....
Average, plow-steel specimens	.....	..	......	....	26 430 000	94 200	246 500	38.2

Single-Wire Test Data.—A summary of test data on the various specimens of single wire tested is given in Table 7. On the basis of these results, an average ultimate strength of cast steel used in these ropes was selected as 219 000 lb per sq in., and for plow steel as 246 500 lb per sq in. A modulus of elasticity of 26 500 000 lb per sq in. was selected as characteristic of both the cast-steel and plow-steel specimens. Fig. 10 is an example of the average stress-strain relation for the ten samples of cast-steel wire 0.105 in. in diameter. As an example of the consistency of these data, for the plow-steel specimens, all had ultimate strengths within 3.2% of the average value, whereas 95% had values of modulus of elasticity within 7% of the average. The cast steel showed similar consistency with minor exceptions, notably the modulus values for the specimens 0.115 in. in diameter. Although the diameters of all test specimens were measured to 0.0001 in., the nominal diameters were used in computing net areas in all further calculations, on the basis that they represented an average condition of manufacture.

Significance of Results

The most significant results of the tension tests on the various wire ropes studied are contained in the plots for increase in the modulus of elasticity of the rope as a whole with repeated loadings. In the past a value of 12 000 000 lb per sq in. has often been considered the maximum modulus that a wire rope would attain, and this value has frequently been termed conservative when used in bending stress formulas. From the data presented herein it may be noted that one excessive loading up to about 50% of the ultimate load will raise the modulus frequently greater than 14 000 000 lb per sq in., and even with working loads as low as 14% of the ultimate, a definite increase in modulus occurs, although it is probable that the value would never reach as high a figure as when overloaded once. For running ropes, it is considered desirable to have a fairly low modulus, so that the rope can "give" and absorb some of the shocks of sudden starting, whereas for stationary installations the reverse is true. In the case of suspender cables and guy-work for which accurate lengths are needed, the value of several pre-stressings to a fairly high load may be seen very plainly. This procedure has been followed in the past, and values for the modulus greater than 19 000 000 lb per sq in. have been obtained on large suspender cables for suspension bridges.

Of equal significance are the load-stress curves for both tension and bending over sheaves. It has been shown that the bending stress is not increased after the rope is once bent over a sheave. and that thereafter the rope at the tangent point behaves as if it were in pure tension. The point of maximum stress has been shown to be at the tangent point, falling off to a minimum at the top of the sheave due to the frictional forces acting. Probably the true point of maximum stress is just slightly above the tangent point, since it takes a short distance for the bending stresses to come into action and the frictional loss is low at this point. The stress distribution

in the strand itself has been pointed out, and indications are that the inner wires take considerably higher stresses than the outer wires at ordinary working loads. The beneficial effect of Lang lay rope in reducing stresses in the wires both in tension and in bending is notable, the reduction being proportional to the cosine of the angle which the surface wires make with the axis of the rope, in this case, 37 degrees. A disadvantage of this type of cable, however, lies in its greater tendency to kink and untwist, and the ends should always be rigidly fixed against rotation.

None of the formulas dealing with wire ropes takes preforming into account. It is significant that for the ropes tested the summary of ultimate loads which follows shows that the preformed ropes are about 4% to 5% weaker in straight tension due to the process of manufacture. However, they are shown to develop less loss of strength due to bending, in some instances by quite appreciable amounts. The initial stress distribution among the individual wires of a preformed rope does not seem to be greatly improved over a non-preformed rope, but there is some tendency for re-adjustment and equalization of stress to occur more quickly in a preformed specimen. The modulus of elasticity of the preformed ropes seems to run slightly higher than that of the non-preformed types. The chief advantages to the use of preformed ropes seems to be the case in handling, cutting, and splicing them, the elimination of kinking to a large extent, and the manner in which they tend to remain closed when several wires are broken rather than bristling with jagged ends.

The curves for loss of strength over sheaves are of primary importance, and a new and simpler method of plotting these curves has been shown. The various features of these curves have been discussed previously, but it is worth noting here that Mr. Rairden's (1) curve based on tests of $\frac{5}{8}$-in. wire ropes does not agree with the results herein obtained, although a ratio of the diameters has been used. This leads to the speculation that possibly the ratio of rope to sheave diameter may not be the proper one to use in such a plot, and indicates the need of further experimentation on ropes of different diameters to discover whether results on one diameter can be transformed to another diameter by a simple ratio.

Summary

Three tables are presented to summarize the wire-rope test results and the formulas with which they were compared. The first, Table 8, presents seven bending-stress formulas, and observed values obtained in most cases by three different methods, as explained previously. Values that are underlined exceed the ultimate strength of the wire, even with no tensile load applied. The second, Table 9, summarizes the observed values of ultimate load and efficiency in tension as well as those predicted by the three formulas. The third, Table 10, presents similar predicted values of the ultimate load in bending over each of the four sheave sizes by two formulas, and the observed values for comparison.

TABLE 8.—SUMMARY OF BENDING-STRESS FORMULAS (ALL STRESSES IN THOUSANDS OF POUNDS PER SQUARE INCH)

Set No. (see Table 2): (1)	Equation (1) (2)	Equation (2) (3)	Equation (3) (4)	Equation (4) (5)	Equation (5) (6)	Equation (6) (7)	Equation (8) (8)	OBSERVED VALUES: Loss of strength divided by A_e (9)	From load-stress curve at breaking point (10)	From plain bending test (tension values) (11)
					(*a*) 18-INCH SHEAVE					
1.....	154.58	139.01	125.01	83.34	68.02	85.66	15.69	11.80	12.3	65.9
2.....	154.58	139.01	125.01	83.34	68.02	94.78	15.69	4.49	14.3	82.1
3.....	154.58	139.01	125.01	83.34	68.02	92.57	14.94	9.00	....	....
4.....	154.58	139.01	125.01	83.34	68.02	81.79	14.94	8.29	5.5	....
9.....	95.69	86.06	77.38	51.59	42.10	47.60	6.91	7.55	19.9	22.5
10.....	95.69	86.06	77.38	51.59	42.10	48.99	6.91	4.52	20.3	28.2
11.....	95.69	86.06	77.38	51.59	42.10	53.03	6.57	5.70*	2.1	....
12.....	95.69	86.06	77.38	51.59	42.10	50.97	6.57	14.78†	12.0†	....
13.....	95.69	86.06	77.38	51.59	42.10	52.20	6.91	15.43	25.0	21.7
					(*b*) 14-INCH SHEAVE					
1.....	198.75	178.73	160.73	107.15	87.45	108.50	19.87	15.57	15.8	81.8
2.....	198.75	178.73	160.73	107.15	87.45	120.05	19.87	10.17	17.6	98.6
3.....	198.75	178.73	160.73	107.15	87.45	117.25	18.92	13.18	13.5	....
4.....	198.75	178.73	160.73	107.15	87.45	103.60	18.92	8.96	7.0	....
9.....	123.04	110.65	99.50	66.34	54.14	60.20	8.75	12.51	21.7	28.4
10.....	123.04	110.65	99.50	66.34	54.14	62.05	8.75	7.64	22.0	36.2
11.....	123.04	110.65	99.50	66.34	54.14	67.17	8.32	8.18*	7.0	....
12.....	123.04	110.65	99.50	66.34	54.14	64.57	8.32	4.80†	5.8†	....
13.....	123.04	110.65	99.50	66.34	54.14	66.13	8.75	17.19	33.5	27.1
					(*c*) 10-INCH SHEAVE					
1.....	278.25	250.22	225.02	150.01	122.43	147.95	27.09	22.95	22.2	104.2
2.....	278.25	250.22	225.02	150.01	122.43	163.70	27.09	13.81	15.1	133.0
3.....	278.25	250.22	225.02	150.01	122.43	159.89	25.80	17.05	15.0	....
4.....	278.25	250.22	225.02	150.01	122.43	141.27	25.80	14.64	15.4	....
9.....	172.25	154.90	139.30	92.87	75.79	82.10	11.94	18.00	23.5	38.4
10.....	172.25	154.90	139.30	92.87	75.79	84.62	11.94	14.81	35.6	50.4
11.....	172.25	154.90	139.30	92.87	75.79	91.59	11.34	10.37*	10.8	....
12.....	172.25	154.90	139.30	92.87	75.79	88.05	11.34	8.99	10.0	....
13.....	172.25	154.90	139.30	92.87	75.79	90.17	11.94	19.54	29.8	35.7
					(*d*) 7-INCH SHEAVE					
1.....	397.50	357.46	321.45	214.30	174.90	203.44	37.25	31.22	28.2	....
2.....	397.50	357.46	321.45	214.30	174.90	225.09	37.25	18.47	24.8	....
3.....	397.50	357.46	321.45	214.30	174.90	219.84	35.48	23.68	15.3	....
4.....	397.50	357.46	321.45	214.30	174.90	194.25	35.48	21.32	23.3	....
9.....	246.07	221.28	198.99	132.66	108.27	112.90	16.41	29.29	31.1	52.5
10.....	246.07	221.28	198.99	132.66	108.27	116.35	16.41	22.45	47.5	72.4
11.....	246.07	221.28	198.99	132.66	108.27	125.94	15.59	17.89*	18.8	....
12.....	246.07	221.28	198.99	132.66	108.27	121.06	15.59	13.94	21.0	....
13.....	246.07	221.28	198.99	132.66	108.27	123.99	16.41	29.50	45.0	47.4

* On basis of ultimate load in tension of 75 000 lb. †Fractured at socket.

Which of the three observed values of bending stress is the most nearly correct is a doubtful question. From the foregoing discussion, the limitations of the plain bending test without tension and the method of extrapolating the load-stress curve to the breaking point are apparent, and the most logical method seems to be the first one presented in Table 8 Column (9)—that of dividing the loss of strength by the product of net area and efficiency in tension. Either on this basis or by striking an average of all three methods,

one must exclude all the formulas except Equation (8) as giving values too conservative for use on stationary ropes. Although Equation (8) frequently does not come very close to the observed values, in view of the doubt as to the accuracy of these latter values, the discrepancy is not nearly as large as for any of the other formulas, and Equation (8), although unwieldy, gives results most comparable with the test data.

TABLE 9.—Summary of Predicted and Observed Values of Ultimate Load in Tension

Set No. (see Table 2):	Equation (10)		Equation (11)	Equation (12)		Observed load, in pounds	Values, efficient (percentage)
	Load, in pounds	Efficient (percentage)		Load, in pounds	Efficient (percentage)		
1..........	68 240	83.2	63 750*	65 530	79.9	68 350	83.3
2..........	68 240	83.2	63 750*	65 530	79.9	64 750	78.9
3..........	68 240	83.2	......	65 530	79.9	69 500	84.7
4..........	68 240	83.2	......	65 530	79.9	67 400	82.2
9..........	77 940	85.2	63 750	73 030	79.9	71 900	78.6
10..........	77 940	85.2	63 750	73 030	79.9	69 800	76.3
11..........	77 940	85.2	......	73 030	79.9	71 650†	78.5†
12..........	77 940	85.2	......	73 030	79.9	72 000	78.7
13..........	87 600	85.2	75 000	82 090	79.9	82 850	80.6

* Using $C = 0.85$ for 6×19 cast-steel ropes.

† Values should be 75 000 lb and 82.1% efficient as explained in test.

For the tension test predictions (Table 9), Equation (11) is admittedly based on minimum values, which are low. Equation (10) gives values that are in most cases slightly too high, whereas Equation (12) fits the test data very well on the average. According to this latter formula, all the ropes tested should show efficiencies of 79.9%, whereas the average for the seven ropes was 80.6 per cent. One factor, however, which the equation does not take into account is the slight loss of strength due to preforming.

TABLE 10.—Summary of Predicted and Observed Values of Ultimate Load in Bending Over Sheaves

Set No. (see Table 2):	Equation (9)*: Sheave Diameter in Inches:				Equation (8): Sheave Diameter, in Inches:				Observed Values: Sheave Diameter, in Inches:			
	18	14	10	7	18	14	10	7	18	14	10	7
1.........	41 630	34 520	22 220	4 940	62 480	60 920	58 220	54 420	64 675	63 500	61 200	58 625
2.........	36 740	29 290	16 400	Minus	58 880	57 320	54 620	50 820	63 425	61 750	60 675	59 300
3.........	40 150	32 330	18 820	Minus	63 910	62 420	59 850	56 230	66 650	65 325	64 100	62 000
4.........	42 270	35 570	23 990	7 700	61 810	60 320	57 750	54 130	64 300	64 050	61 925	59 425
9.........	56 300	52 150	45 000	34 900	69 020	68 250	66 920	65 060	69 425	67 800	66 000	62 300
10.........	54 600	50 400	43 200	33 000	66 920	66 150	64 820	62 960	68 350	67 350	65 050	62 600
11.........	56 920	52 080	43 720	31 960	72 260	71 530	70 270	68 500	73 050	72 200	71 450	68 875
12.........	55 240	50 780	43 070	32 240	69 260	68 530	67 270	65 500	67 150†	70 425†	69 050	67 425
13.........	66 100	61 400	53 200	41 700	79 970	79 200	77 870	76 010	77 600	77 000	76 200	72 800

* In which $s = \frac{E_r\, d}{D + d_r}$. † Fractured at socket.

Ultimate loads in bending could be predicted for any of the formulas listed in Table 8, but the last two only have been selected as giving possibly reasonable values. Mr. Meals states (1) that for his formula (a modification

of Mr. Leffler's formula), the error is a maximum for the lower ratios of $\frac{D}{d_r}$. This statement is fully confirmed and values of ultimate load over the 7-in. sheave are far too low, in two cases even being a minus quantity. For large-sized sheaves as recommended in modern practice, Equation (10) tends to approach the observed values, but is consistently conservative. Equation (12), from which the expression for bending stress was derived, might be expected to give equally good predictions for strength, and observation will disclose that predictions based on this formula vary from the observed values in no case by more than 14%, in this case on the safe side. The average error on the unsafe side in only 2.5% and on the safe side, 4.5 per cent.

It should be remarked that all the formulas that have been found herein to vary from the observed data, have varied on the safe side, and that those which best fit the data vary sometimes on the safe side, but almost as often on the unsafe side, although the percentage error is very small in comparison with all the other formulas. Undoubtedly, velocity and reverse bending affect the ultimate load and the bending stress adversely, and until more tests are made of wire rope in motion under load, it is preferable on such installations to err on the safe side in stress computations.

Conclusions

From a study of the data obtained in this investigation, the following conclusions have been drawn, applying to stationary wire ropes with hemp centers in tension and in bending over sheaves:

(1) The modulus of elasticity of a rope as a whole was increased about 50% by one loading to 50% of the ultimate load, and continued to rise slowly upon further repetitions of the load. Even for ordinary working loads such a rise took place, but the values for modulus were only from 60 to 70% of the values when overloaded by pre-stressing.

(2) For tension tests of regular lay ropes below the proportional limit of the wires, the stress in the outer wires coincided very closely with that obtained by dividing the load by the net area of cross-section.

(3) The variation in stress with load for ropes bent over sheaves was exactly the same as for the same ropes in tension, except that a definite bending stress, the magnitude of which depended on the sheave size, was added at the time of bending, and this did not vary as the load increased.

(4) The maximum stress in a wire rope bent over a sheave occurred at or immediately above the point of tangency to the sheave, and the rope might be expected to fracture at this point. The minimum stress in the rope occurred at the top of the sheave.

(5) Initial fracture in ropes of 6 × 19 construction, and probably also in the case of those of 6 × 7 construction occurred in the interior wires of the strands, the stress in the core wire being always greater than that in the outer wires up to the point of initial fracture.

(6) The percentage loss of strength of a rope bent over a sheave varied linearly with the ratio of rope diameter to sheave diameter at the root.

(7) The coefficient of friction of a regular lay rope on a steel sheave was roughly 0.14, and of a Lang lay rope, roughly, 0.38.

(8) The stresses in the outer wires of a Lang lay rope were reduced in proportion to the cosine of the angle of inclination with the axis of the rope, a reduction of very nearly 20% for ordinary construction.

(9) A preformed rope showed less loss of strength in bending over sheaves, but also about 5% lower tensile strength and efficiency than a non-preformed rope.

(10) The most satisfactory formulas found for the prediction of bending stress, tensile strength, and loss of strength due to bending, were those presented by Mr. Carstarphen (1).

Acknowledgments

The testing program was conducted as a co-operative investigation with the Wickwire Spencer Steel Company, which Company furnished all the wire ropes tested and fitted the sockets. The tests were made in the Fritz Engineering Laboratory of Lehigh University, at Bethlehem, Pa., between November 1934, and May, 1935. The writer is indebted to Messrs. A. S. Rairden and Carl King, of the Wickwire Spencer Steel Company, for their valuable aid and suggestions, and to Inge Lyse, M. Am. Soc. C. E., Research Associate Professor of Engineering Materials, for his aid in supervising the tests and interpreting the results. Acknowledgment is also made to the members of the Laboratory Research Staff who have assisted materially in the conducting of these tests. A number of the illustrations used have been loaned by Mr. Rairden.

APPENDIX I

Notation

The symbols used in this paper are defined as follows:

a = angle that a helical wire makes with the axis of the strand; a_i = Angle a of the wires in the ith layer; a_n = angle of lay of Wire No. n;

b = angle that a strand makes with the axis of the rope;

d = diameter of a wire in a rope; d_r = diameter of a wire rope;

e = base of Naperian logarithms;

f = coefficient of friction;

g = a subscript denoting "at the point of tangency";

i = a subscript denoting the ith layer of strands in a rope;

l = a subscript denoting "layers";

n = number; n_s = number of strands in a rope; n_w = number of wires in a given diameter in the ith layer; n_l = number of layers of wires in a strand; as a subscript, n denotes "number";

p = a subscript denoting "at the top";

r = radius of a wire in a rope; r_s = radius from the center of the strand to the center of the wire in question; r_r = radius from the center of a wire rope to the center wire of a strand; as a subscript, r denotes "rope";
s = unit bending stress; as a subscript, s denotes "strand";
t = ultimate unit tensile strength of a wire; as a subscript, t denotes "tension";
w = a subscript denoting "wire";
A = area of a wire rope;
C = a constant representing a relation between the total tension on a wire rope, and its diameter, for various constructions;
D = diameter of sheave;
E = modulus of elasticity of a wire in a rope; E_r = modulus of elasticity of a wire rope;
G = modulus of rigidity in a rope $= \frac{E}{2}(1 + \mu)$;
P = loss of strength in a wire due to bending;
R = radius of a sheave $= \frac{1}{2}(D + d_r)$;
S = ultimate strength of a wire rope in bending; S_i = strength of a wire in the ith layer; S_t = strength of a wire rope in tension; S_w = ultimate strength of a wire;
α = angle between the perpendicular to the axis of a rope and the tangent to the center line of a wire;
ε = efficiency of a rope in plain tension;
μ = Poisson's ratio.

APPENDIX II

Bibliography

(1) **Effect of Bending Wire Rope,** by F. C. Carstarphen, M. Am. Soc. C. E., *Transactions,* Am. Soc. C. E., Vol. 98 (1933), p. 562. Discussion by Messrs. C. D. Meals, A. S. Rairden, F. W. Deck, and others.

(2) **Determination of Stresses in Wire Rope as Applied to Modern Engineering Problems,** by James F. Howe, *Transactions,* A. S. M. E., Vol. 40 (1918), p. 1043.

(3) **Heavy Duty Wire Ropes and Sheaves,** by B. R. Leffler, M. Am. Soc. C. E., *Civil Engineering,* Vol. 1, No. 2, p. 107.

(4) **Six Formulas Compared,** by the late R. C. Strachan, M. Am. Soc. C. E., *Civil Engineering,* Vol. 1, No. 2, p. 111.

(5) **R. W. Chapman,** in *Engineering Review,* October, 1908, Vol. 19, p. 258.

(6) **Strength and Other Properties of Wire Rope,** by J. H. Griffith, M. Am. Soc. C. E., and J. E. Bragg, *Technological Paper No. 121,* National Bureau of Standards.

(7) **Bending Stresses in Wire Rope**, by C. D. Meals, Assoc. M. Am. Soc. C. E., *Transactions*, A. S. M. E., Vol. 51 (1929) II MH-51-5-21.

(8) **Acceleration Stresses in Wire Hoisting Ropes**, by G. P. Boomsliter, M. Am. Soc. C. E., *Transactions*, Am. Inst. of Min. and Met. Engrs., Vol. LXXV (1927), p. 74.

(9) **Some Tests of Steel Wire Rope on Sheaves**, by Edward Skillman, *Technological Paper No. 229*, National Bureau of Standards, Vol. 17.

(10) **The Testing of Rope Wire and Wire Rope**, by A. V. deForest and L. W. Hopkins, *Proceedings*, A. S. T. M., Vol. 32 (1932), Pt. II, p. 398.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
PAPERS

VARIED FLOW IN OPEN CHANNELS OF ADVERSE SLOPE

By Arthur E. Matzke[1], Jun. Am. Soc. C. E.

Synopsis

A canal may be said to possess an adverse slope, if its bottom rises in the direction of the flow. When, as in the usual case, the bottom falls in the direction of flow, a channel may be said to possess a sustaining slope. An example of a layout with an adverse slope is a canal connecting a tidal basin or an area to be drained with the sea. Another case is the approach to a low spillway with a bed gently sloping over a considerable length. Although cases of this kind occur rather infrequently, there are instances in which a knowledge of the flow and methods of numerical computations are essential.

The general equations for varied flow in channels of adverse slope were originally developed by B. A. Bakhmeteff, M. Am. Soc. C. E., about 1910, the method of treatment being in general similar to that which he presented[2] in 1932. Practical application in engineering design, however, depends on knowing the numerical values of a function, similar to the "varied flow function", for canals of sustaining slopes.[3] The required function values were determined by graphical integration and are offered in the form of appropriate tables, together with a brief survey of the underlying theory and some numerical examples making clear the methods of practical applications.

General Equation of Flow in Canals of Adverse Slope

In general, the reasoning leading to the establishment of the general equation of flow in canals of adverse slope follows that developed by Professor Bakhmeteff[4] for the case of sustaining slope. Since the derivation of the varied flow equation for canals of adverse slope is practically unknown, an outline of the development will be presented herein. The symbols used are defined as they occur and are summarized in the Appendix for convenience of reference.

Note.—Discussion on this paper will be closed in May, 1936, *Proceedings*.

[1] Research Asst., Dept. of Civ. Eng., Columbia Univ., New York, N. Y.

[2] "Hydraulics of Open Channels", Eng. Societies Monograph, McGraw-Hill Co., 1932.

[3] *Loc. cit.*, pp. 308–311.

[4] *Loc. cit.*, Paragraph 12.

With reference to Fig. 1(*a*), a discharge, Q, in cubic feet per second, is assumed to flow in a canal of given cross-section. The adverse bottom slope is denoted by the negative sign, thus, $-s_0$. The discharge is referred to a parametric depth value, y_0, which is the depth of uniform flow for the same

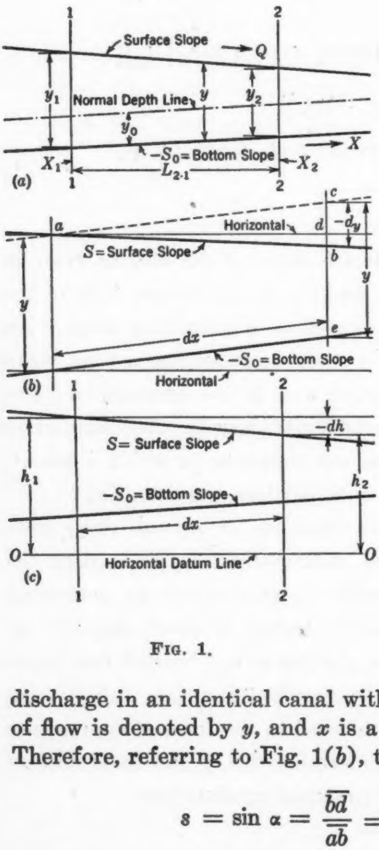


FIG. 1.

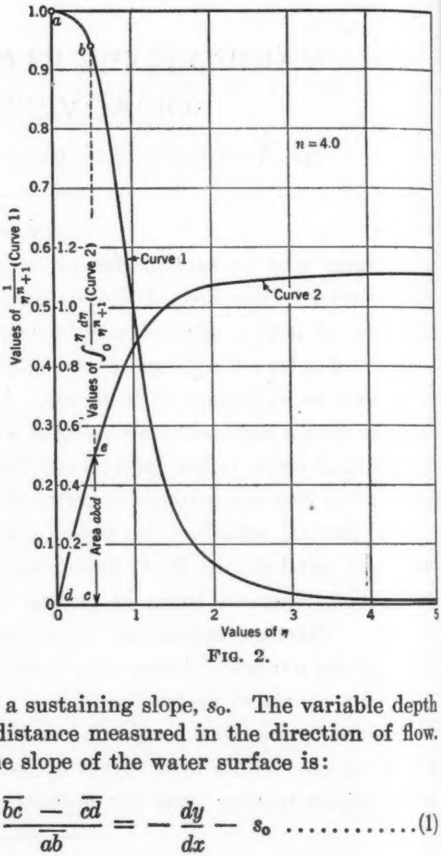


FIG. 2.

discharge in an identical canal with a sustaining slope, s_0. The variable depth of flow is denoted by y, and x is a distance measured in the direction of flow. Therefore, referring to Fig. 1(*b*), the slope of the water surface is:

$$s = \sin \alpha = \frac{\overline{bd}}{\overline{ab}} = \frac{\overline{bc} - \overline{cd}}{\overline{ab}} = -\frac{dy}{dx} - s_0 \quad \text{............(1)}$$

Referring to Fig. 1(*c*) and bearing Equation (1) in mind, the loss of energy head, de_r, over the distance, dx, becomes[4]:

$$\frac{de_r}{dx} = -\left(s_0 + \frac{dy}{dx} + \frac{d}{dx}\left(\frac{V^2}{2\,g}\right)\right) \quad \text{................(2)}$$

Whereas[5], $\frac{de_r}{dx} = \frac{Q^2}{K^2(y)}$, in which $K(y)$ is the value of the conveyance or carrying capacity of a given canal cross-section for any surface depth, y, and[6]

[5] "Hydraulics of Open Channels", by B. A. Bakhmeteff, M. Am. Soc. C. E., Eng. Societies Monograph, McGraw-Hill Co., 1932, Equation (14), p. 26.

[6] *Loc. cit.*, Equation (20), p. 31.

$\frac{d}{dx}\left(\frac{V^2}{2\,g}\right) = -\frac{Q^2}{g} \times \frac{b}{a^3} \times \frac{dy}{dx}$ in which a is the cross-sectional area of flow and b is the top width of the cross-sectional area of flow. Therefore, Equation (2) becomes:

$$-\,s_0 - \frac{dy}{dx} = \frac{Q^2}{K^2\,(y)} - \frac{Q^2}{g} \times \frac{b}{a^3} \times \frac{dy}{dx} \quad \ldots\ldots\ldots\ldots(3)$$

Using the parametric depth, y_0[7], explained previously, and the hydraulic exponent, n[8], it follows that $\frac{Q^2}{K^2\,(y)} = s_0 \left(\frac{y_0}{y}\right)^n$. Furthermore, as $\frac{Q^2}{g} \times \frac{b}{a^3}$ in Equation (3) may be expressed as being equal to $\beta\,\frac{K^2\,(y_0)}{K^2\,(y)}$ in which $\beta = \frac{s_0}{s_c}$[9] and s_c[10] is the critical slope at the varying depth, y, Equation (3) may be written as follows:

$$\frac{dy}{dx} = -\,s_0\,\frac{1 + \left(\frac{K_0}{K}\right)^2}{1 - \beta\left(\frac{K_0}{K}\right)^2} = -\,s_0\,\frac{1 + \left(\frac{y_0}{y}\right)^n}{1 - \beta\left(\frac{y_0}{y}\right)^n} \quad \ldots\ldots\ldots\ldots(4)$$

Equation (4) is the general differential equation of varied flow in canals with adverse slope. Separating the variables and designating $\eta = \frac{y}{y_0}$ (that is, $dy = y_0\,d\,\eta$), Equation (4) becomes:

$$\frac{s_0}{y_0}\,dx = -\,d\,\eta + (1 + \beta)\,\frac{d\,\eta}{\eta^n + 1} \quad \ldots\ldots\ldots\ldots\ldots\ldots(5)$$

The distance, $x_2 - x_1 = l_{2\text{-}1}$, between two sections with the respective depths, y_2 and y_1, which corresponds to $\eta_2 = \frac{y_2}{y_0}$ and $\eta_1 = \frac{y_1}{y_0}$, is obtained by integrating Equation (5); thus:

$$\frac{s_0}{y_0}\left(x_2 - x_1\right) = \frac{s_0}{y_0}\,l_{2-1} = -\,(\eta_2 - \eta_1) + \int_{\eta_1}^{\eta_2} (1 + \beta)\,\frac{d\,\eta}{\eta^n + 1} \quad \ldots(6)$$

Assuming that the value of β may be taken as a constant average value for the range of integration, and designating $\int_0^{\eta} \frac{d\,\eta}{\eta^n + 1} = B'\,(\eta)$ (the "varied flow function" for canals of adverse slope) Equation (6) becomes:

$$l_{2\text{-}1} = \frac{y_0}{s_0}\left[-\,(\eta_2 - \eta_1) + (1 + \beta)\left\{B'\,(\eta_2) - B'\,(\eta_1)\right\}\right] \quad \ldots\ldots(7)$$

which is the general equation of varied flow for channels of adverse slope.

---

[7] "Hydraulics of Open Channels", by B. A. Bakhmeteff, M. Am. Soc. C. E., Eng. Societies Monograph, 1932, p. 22.

[8] *Loc. cit.*, p. 84.

[9] *Loc. cit.*, p. 52.

[10] *Loc. cit.*, p. 47.

For comparison, in the case of sustaining bottom slope:

$$s = s_0 - \frac{dy}{dx} \quad \text{.........................(8)}$$

the varied flow equation is[11]:

$$l_{2-1} = \frac{y_0}{s_0}\left\{(\eta_2 - \eta_1) - (1 - \beta)\left[B(\eta_2) - B(\eta_1)\right]\right\} \quad \text{.......(9)}$$

In other words, the formulas for adverse flow (Equations (1) and (7)) differ from those for sustaining flow (Equations (8) and (9)) only in the signs and in the form of the varied flow function, the essential structure of both being similar.

Integration Procedure

For practical computations it is imperative to know the value of the varied flow function,

$$B'(\eta) = \int_0^\eta \frac{d\eta}{\eta^n + 1} \quad \text{....................(10)}$$

within the practical range of the hydraulic exponents, which usually lies between $n = 3$ and $n = 4$. For the particular value of $n = 3$ and $n = 4$, $B'(\eta)$ may be obtained analytically. The forms of the respective quadratures are:

For $n = 3$, substitute $z = -\eta$ so that $\int \frac{d\eta}{\eta^3 + 1} = \int \frac{dz}{z^3 - 1}$. Then,

$$\int \frac{dz}{z^3 - 1} = \frac{1}{6} \log_e \frac{(z-1)^2}{z^2 + z + 1} + \frac{1}{\sqrt{3}} \operatorname{arccot} \frac{2z + 1}{\sqrt{3}} \quad \text{......(11)}$$

For $n = 4$,

$$\int \frac{d\eta}{\eta^4 + 1} = \frac{1}{4\sqrt{2}} \log_e \left(\frac{\eta^2 + \eta\sqrt{2} + 1}{\eta^2 - \eta\sqrt{2} + 1}\right) + \frac{1}{2\sqrt{2}} \arctan\left\{2\sqrt{2} + 1\right\}$$

$$+ \arctan\left\{2\sqrt{2} - 1\right\} \quad \text{....................(12)}$$

For intermediate values of n, one could take recourse to computation by series, but this is a laborious and lengthy procedure scarcely warranted by the character of the problem. It seemed that results sufficiently accurate for engineering practice might be obtained by graphical integration. The method is illustrated by Fig. 2 which gives the curves for the particular case of $n = 4$.

For values of η, the corresponding values of $\frac{1}{\eta^n + 1}$ are found, which when plotted against η give Curve 1.

[11] "Hydraulics of Open Channels", by B. A. Bakhmeteff, M. Am. Soc. C. E., Eng. Societies Monograph, 1932, Equation (84), p. 87.

For each η-ordinate, the value of $\int_0^{\eta} \frac{d\eta_0}{\eta^n + 1}$ is the numerical value of the area under the curve to the left of that ordinate. These areas may be computed by a planimeter or simply by counting squares. Curve 2 is drawn so as to represent for each η-ordinate, the corresponding area to an appropriate scale as Ordinate ce = Area $abcd$. The writer compared values of $B'(\eta)$

TABLE 1.—COMPARISON OF GRAPHICAL AND ANALYTICAL VALUES OF INTEGRAL

$$\int_0^{\eta} \frac{d\,\eta}{\eta^n + 1}$$

Values of η	VALUES OF THE INTEGRAL		Percentage of error
	Graphical	Analytical	
0.500.............	0.4999	0.4940	—0.02
2.500.............	1.0880	1.0898	—0.16
5.000.............	1.1045	1.1081	—0.33

determined graphically for $n = 3$ and $n = 4$ with those computed analytically by substituting numerical values for η in Equation (11) and Equation (12), respectively (see Table 1). In no case did the discrepancy exceed 0.33 per cent.

TABLE 2.—VALUES OF THE VARIED FLOW FUNCTION, $B'(\eta)$, FOR CANALS OF ADVERSE SLOPE

Values of η	HYDRAULIC EXPONENTS, n:						Values of η	HYDRAULIC EXPONENTS, n:					
	3.0	3.2	3.4	3.6	3.8	4.0		3.0	3.2	3.4	3.6	3.8	4.0
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(1)	(2)	(3)	(4)	(5)	(6)	(7)
0.00	0.000	0.000	0.000	0.000	0.000	0.000	1.80	1.064	1.062	1.061	1.060	1.059	1.056
0.05	0.050	0.050	0.050	0.050	0.050	0.050	1.85	1.071	1.068	1.066	1.064	1.062	1.060
0.10	0.100	0.100	0.100	0.100	0,100	0 100	1.90	1.078	1.075	1.072	1.069	1.066	1.063
0.15	0.150	0.150	0.150	0.150	0.150	0.150	1.95	1.083	1.080	1.077	1.073	1.070	1.066
0.20	0.200	0.200	0.200	0.200	0.200	0.200	2.00	1.090	1.086	1.083	1.076	1.074	1.070
0.25	0.249	0.249	0.249	0.250	0.250	0.250	2.10	1.100	1.095	1.090	1.082	1.079	1.075
0.30	0.298	0.299	0.299	0.300	0.301	0.302	2.20	1.109	1.102	1.097	1.088	1.085	1.080
0.35	0.345	0.346	0.347	0.348	0.349	0.350	2.30	1.117	1.110	1.102	1.093	1.090	1.084
0.40	0.393	0.394	0.396	0.397	0.398	0.399	2.40	1.124	1.116	1.108	1.097	1.094	1.087
0.45	0.439	0.440	0.441	0.443	0.445	0.447	2.50	1.131	1.121	1.113	1.101	1.097	1.090
0.50	0.485	0.488	0.490	0.491	0.493	0.494	2.60	1.136	1.126	1.116	1.104	1.100	1.092
0.55	0.529	0.532	0.535	0.537	0.539	0.540	2.70	1.141	1.130	1.120	1.108	1.102	1.094
0.60	0.572	0.575	0.578	0.581	0.584	0.586	2.80	1.146	1.134	1.124	1.110	1.104	1.096
0.65	0.611	0.615	0.620	0.624	0.628	0.630	2.90	1.150	1.137	1.126	1.112	1.106	1.097
0.70	0.650	0.655	0.661	0.665	0.668	0.672	3.00	1.154	1.140	1.128	1.114	1.108	1.099
0.75	0.685	0.692	0.697	0.700	0.706	0.709	3.10	1.158	1.143	1.131	1.116	1.109	1.100
0.80	0.719	0.726	0.732	0.737	0.742	0.746	3.20	1.161	1.145	1.133	1.118	1.110	1.100
0.85	0.752	0.758	0.764	0.770	0.775	0.780	3.30	1.164	1.147	1.134	1.119	1.111	1.101
0.90	0.782	0.789	0.796	0.802	0.807	0.812	3.40	1.166	1.150	1.136	1.121	1.112	1.102
0.95	0.809	0.816	0.822	0.829	0.835	0.840	3.50	1.169	1.151	1.137	1.122	1.113	1.103
1.00	0.836	0.843	0.850	0.854	0.862	0.866	3.60	1.171	1.153	1.139	1.123	1.114	1.104
1.05	0.859	0.867	0.873	0.878	0.885	0.890	3.70	1.173	1.155	1.140	1.124	1.115	1.104
1.10	0.881	0.889	0.896	0.901	0.907	0.911	3.80	1.175	1.156	1.141	1.125	1.116	1.105
1.15	0.901	0.908	0.915	0.918	0.925	0.928	3.90	1.177	1.157	1.142	1.126	1.116	1.105
1.20	0.920	0.927	0.935	0.938	0.944	0.948	4.00	1.178	1.159	1.143	1.126	1.117	1.105
1.25	0.938	0.945	0.950	0.955	0.958	0.963	4.10	1.180	1.160	1.144	1.127	1.118	1.106
1.30	0.955	0.960	0.964	0.968	0.973	0.976	4.20	1.181	1.161	1.145	1.128	1.118	1.107
1.35	0.971	0.975	0.978	0.982	0 986	0.989	4.30	1.183	1.162	1.145	1.128	1.119	1.107
1.40	0.985	0.988	0.991	0.994	0.997	1,000	4.40	1.184	1.163	1.146	1.129	1.119	1.107
1.45	0.997	1.000	1.003	1.006	1.008	1.010	4.50	1.185	1.164	1.147	1.129	1.119	1.107
1.50	1.010	1.012	1.014	1.015	1.017	1.019	4.60	1.186	1.165	1.147	1.130	1.120	1.108
1.55	1.020	1.022	1.023	1.024	1.025	1.027	4.70	1.187	1.165	1.148	1.130	1.120	1.108
1.60	1.028	1.030	1.031	1.032	1.033	1.034	4.80	1.188	1.166	1.148	1.130	1.120	1.108
1.65	1.038	1.039	1.040	1.040	1.041	1.041	4.90	1.188	1.167	1.149	1.131	1.120	1.108
1.70	1.048	1.048	1.047	1.047	1.046	1.046	5.00	1.189	1.167	1.149	1.131	1.120	1.108
1.75	1.057	1.056	1.055	1.054	1.053	1.052	...	....	....	....	....	....	....

Curves were made then for intermediary exponents at intervals of 0.2 and the results summarized in Table 2. Values of $B'(\eta)$ for exponents between the tabular values can either be taken by interpolation or, if desired, in each particular case, a special curve of $\dfrac{1}{\eta^n + 1}$ and $\displaystyle\int_0^\eta \frac{d\eta}{\eta^n + 1}$ may be computed and traced.

Such curves and tabular values allow a simple and rapid solution of practical problems. With reference to Fig. 1(*a*) the distance, $l_{2\text{-}1}$, between two selected depths, with $\eta_2 = \dfrac{y_2}{y_0}$ and $\eta_1 = \dfrac{y_1}{y_0}$, is determined from Equation (2) by inserting the values of $B'(\eta_2)$ and $B'(\eta_1)$ from Table 2.

Types of Surface Curves

Due to the direction of the slope the motion in the canal takes place at the expense of the specific energy available in the initial section, which is dissipated in the course of the movement. Only two types of surface curves are possible: (1) For depths above the critical ($y > y_c$), a drop curve (see Fig. 3) of the M_2-type[12] which is characterized by a decreasing depth, y, termi-

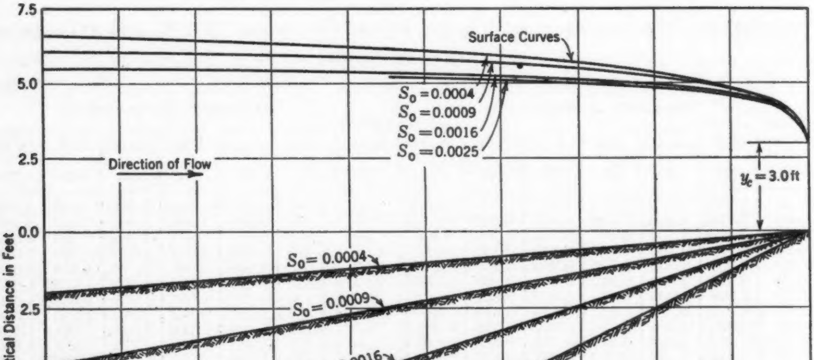


FIG. 3.

nating at y_c; and (2) in the range of depths below the critical ($y < y_c$), a rising curve (of the M_3-type)[13] illustrated by Fig. 4 and also terminating at y_c.

[12] "Hydraulics of Open Channels", by B. A. Bakhmeteff, M. Am. Soc. C. E., Eng. Societies Monograph, 1932, Fig. 60, p. 76.

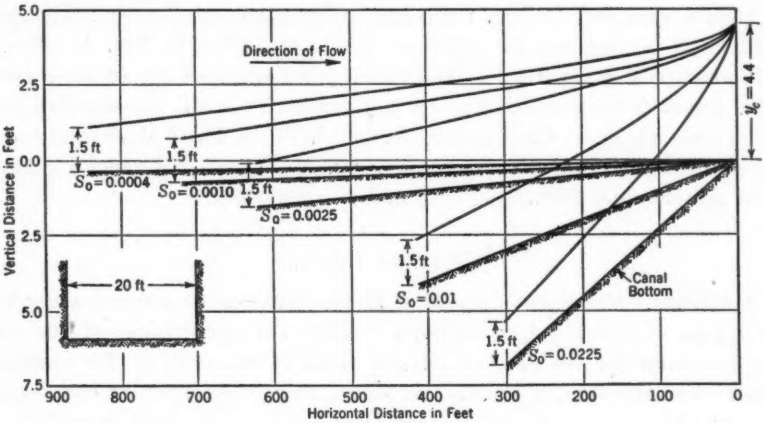


FIG. 4.

PRACTICAL EXAMPLES

Example 1.—M'_2-Curve.—Assuming a discharge of $Q = 519$ cu ft per sec, and a slope, $s_0 = 0.0004$, following the usual procedure, the value of $y_0 = 6.56$ ft[13]. The critical depth is $y_c = 3$ ft. The hydraulic exponent is taken at $n = 3.80$; and the average value of β is 0.06. At the point of critical depth, $\eta_c = \dfrac{3}{6.56} = 0.457$, and, by interpolation, from Table 2, $B'(\eta_c) = 0.452$.

Calculating the distance from $y_c = y_2$ to a point at which the depth is $y_1 = 5.0$ ft, with $\eta_1 = \dfrac{5.0}{6.56} = 0.762$ and B' $(0.762) = 0.715$, by Equation (7) $l_{2\text{-}1} = \dfrac{6.56}{0.0004}$ $[-(0.457 - 0.762) + 1.06\ (0.452 - 0.715)] = 459$.

A similar procedure determines the respective distance from $y_c = 3$ ft to a section with $y_1 = 4$ ft, 6 ft, etc. Thus, the entire surface curve may be traced. Fig. 3 gives the curves for the same discharge and cross-section, but for different values of the adverse slope, ranging from -0.0004 to -0.0025.

Example 2.—M'_3-Curve (Fig. 4).—Assume a discharge of $Q = 1\,080$ cu ft per sec, and $y_c = 4.4$ ft[14]; and, with $-s_0 = 0.0004$, $y_0 = 8.65$ ft. Furthermore, let $n = 3$ and $1 + \beta = 1.17$; and assume that a sluice-gate maintains an initial depth, $y_1 = 1.5$ ft. The rising curve is to be computed from these data. To illustrate, determine the distance from the section with $y_1 = 3.0$ ft to a section with $y_2 = 3.50$ ft; thus: $\eta_1 = \dfrac{3.0}{8.65} = 0.347$; and $\eta_2 = \dfrac{3.50}{8.65} = 0.405$. Accordingly, from Table 2, $B'(\eta_1) = 0.342$; and $B'(\eta_2) = 0.398$. Substituting these values into Equation (7),

$$l_{2\text{-}1} = \frac{8.65}{0.0004}\ [-(0.405 - 0.347) + 1.17\ (0.398 - 0.342)] = 160 \text{ ft}$$

[13] "Hydraulics of Open Channels", by B. A. Bakhmeteff, M. Am. Soc. C. E., Eng. Societies Monograph, 1932, Canal Type *D*, p. 321.

[14] *Loc. cit.*, p. 320, Canal Type *C*.

Curves computed in the foregoing manner for a constant discharge of 1 080 cu ft per sec but for different slopes, are given in Fig. 4. The end of the curves is $y_c = 4.4$ ft. The rather paradoxical aspect of these curves, which depict the characteristics of water moving up a hill, is controlled partly by the scale adopted. Experiments were made in the Fluid Mechanics Laboratory of Columbia University, in New York, N. Y., and the general character of the movement as illustrated by Fig. 4 was confirmed.

Delivery Curves

A delivery curve shows the change in the discharge in a canal under varying depths at certain given sections—usually the extremities—of the canal. The reasoning for the case of adverse slope is the same as for sustaining slope[15]. For the sake of brevity, therefore, only one particular case, that of delivery with y_2 constant will be treated to supply a practical example.

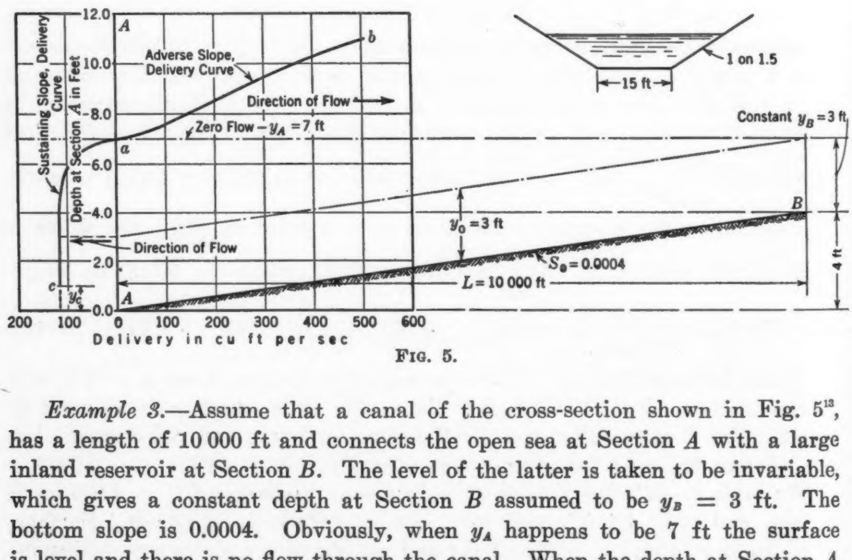


FIG. 5.

Example 3.—Assume that a canal of the cross-section shown in Fig. 5[13], has a length of 10 000 ft and connects the open sea at Section A with a large inland reservoir at Section B. The level of the latter is taken to be invariable, which gives a constant depth at Section B assumed to be $y_B = 3$ ft. The bottom slope is 0.0004. Obviously, when y_A happens to be 7 ft the surface is level and there is no flow through the canal. When the depth at Section A exceeds 7 ft there will be flow from Section A to Section B, the motion taking place against an adverse slope. A delivery curve (Curve ab, Fig. 5) indicates the value of the discharge, Q, for corresponding values of the variable depth, y_A.

To find a point of the delivery curve assume $y_0 = 5$ ft, which with K (5 ft) $= 15\,200$, corresponds to a discharge, $Q = K$ (5 ft) $\sqrt{s_0} = 304$ cu ft per sec. Assume that $1 + \beta = 1.06$ and $n = 3.80$. The problem now is reduced to that of Fig. 3, namely, to find the depth, $y_A = y_1$, which, with

[15] "Hydraulics of Open Channels", by B. A. Bakhmeteff, M. Am. Soc. C. E., Eng. Societies Monograph, 1932, p. 143.

$l_{2-1} = 10\,000$ and $y_0 = 5$ ft, will correspond to $y_2 = y_B = 3$ ft, as given. Equation (7) may be rewritten in the form:

$$l_{2-1} \frac{s_0}{y_0} = [-\eta_2 + (1+\beta) B'(\eta_2)] - [-\eta_1 + (1+\beta) B'(\eta_1)]$$
$$= z'(\eta_2) - z'(\eta_1) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

in which $z'(\eta)$ is a function of η of the form,

$$z'(\eta) = -\eta + (1+\beta) B'(\eta) \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

In the present case, with y_2 (and thus η_2) given, the only unknown in Equation (13) is $\eta = \frac{y_1}{y_0}$ implicitly contained in the function, $z'(\eta_1)$.

For $y_0 = 5$ ft, therefore, $l_{2-1} = \frac{s_0}{y_0} = 10\,000 \times \frac{4 \times 10^{-4}}{5} = 0.8$; $\eta_2 = \frac{3}{5} = 0.60$; and $B'(\eta_2) = 0.584$. Consequently, by Equation (14), $z'(\eta_2) = -0.600 + 1.06 \times 0.584 = 0.019$. The solution of the problem involves finding the value of $\eta_1 = \frac{y_A}{y_0}$ which satisfies the equation:

$$z'(\eta_1) = z'(\eta_2) - l_{2-1} \frac{s_0}{y_0} = -0.781 \ldots\ldots\ldots\ldots (15)$$

The most practical way to solve Equation (14) is to build an auxiliary curve of $z'(\eta)$ plotted against η.

The value of η_1 which is sought is 1.912 and, thus, $y_A = \eta_1 \times y_0 = 1.912 \times 5 = 9.56$ ft. Hence, a point of the delivery curve is established with $y = 9.56$ ft and the corresponding discharge of 304 cu ft per sec. By assigning other values to the normal depth, y_0, other points of the delivery curve may be obtained, resulting in the curve, *ab* (Fig. 5).

When the depth at Section *A* becomes less than 7 ft, the flow will be in the direction of the sustaining slope. The corresponding delivery curve is *ac*.

Conclusions

The case of flow in channels of adverse slope completes the scope of application of the Belanger equation of varied flow based on the assumption of parallel flow to practical engineering design. It is hoped that the varied flow equation (Equation (7)) and the tables of values of the varied flow function will provide a direct and convenient method of solving some of the hydraulic problems connected with the design of open channels of adverse slope.

Acknowledgments

The writer wishes to express his sincere appreciation to Boris A. Bakhmeteff, M. Am. Soc. C. E., for aid and guidance during the course of the work.

APPENDIX

NOTATION

c = a subscript denoting "critical";
l = length; distance between two given sections; $l_{2\text{-}1}$ = distance between Section 2 and Section 1;
n = an hydraulic exponent;
s = slope of the water surface; s_0 = sustaining slope of a channel bottom; $-s$ = adverse slope of a channel bottom; s_c = critical slope of water surface;
x = a variable distance measured in the direction of flow;
y = a parameter denoting variable depth of flow; y_0 = normal depth, or the depth in the case of uniform movement; y_c = critical depth;
z = a function; $z'(\eta)$ = a function of η referred to the case of adverse slope;
A = area;
B = a function; $B(\eta)$ = the varied flow function $= \int_0^\eta \frac{d\eta}{\eta^n - 1}$, for sustaining slope; $B'(\eta)$ = varied flow factor for adverse slope;
M = a type of surface curve: M_2-curve, M_3-curve, etc.;
Q = rate of discharge, or flow;
V = average velocity in a section;
α = slope angle $= \sin^{-1} s$;
β = the relation between the bottom slope, s_0, and the critical slope, s_c;
η = the relation of the varying depth of flow, y, to the normal depth, y_0 $\left(\text{that is, } \eta = \frac{y}{y_0}\right)$.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

REPORTS

FLOOD PROTECTION DATA[1]

PROGRESS REPORT OF THE COMMITTEE

The activities of the Committee on Flood Protection Data, during 1935, have been centered on acting in an advisory capacity to the Water Planning Committee of the National Resources Board in connection with the compilation and analyses of flood data undertaken with an allotment of funds disbursed by the Public Works Administration. The data were compiled, analyzed, and prepared by the Water Resources Branch of the United States Geological Survey under the general direction of N. C. Grover, M. Am. Soc. C. E., Chief Hydraulic Engineer, U. S. Geological Survey, assisted by R. W. Davenport, M. Am. Soc. C. E., Hydraulic Engineer in Charge of the Division of Water Utilization, and under the detailed direction of C. S. Jarvis, M. Am. Soc. C. E., Senior Engineer. This work had engaged the attention of the Committee during the year preceding, the Committee having been appointed by the Board of Direction of the Society originally for the purpose of assisting the Mississippi Valley Committee of the Federal Government which, early in 1934, initiated the aforementioned flood-data compilations and studies. The results of the latter have recently appeared in published form as U. S. Geological Survey *Water Supply Paper 771*, entitled: "Floods in the United States—Magnitude and Frequency." The labors of the Committee in connection with this work, therefore, have come to a close.

In comment on this report it should be stated that limitations imposed by restricted funds, at the outset, made it appear impracticable to compile and publish more than a part of the great bulk of flood data available. The volume, therefore, in no sense claims to be a compendium of flood information for all streams of the United States. Logically, it should be viewed as probably the initial volume of what may become a series of Government reports on flood data. Despite its limitations, the report represents the fruit of a small allotment of public funds well spent, and should prove of much practical value to engineers as well as to public officials concerned with flood problems. In addition to chronologically arranged tabulations of flood events for about 200 river stations, it contains chapters devoted to flood frequency analysis, the unit-hydrograph method of determining flood run-off from rainfall, and the flood characteristics of various regions of the United States. It marks

NOTE.—Discussion on this report will be closed in May, 1936, *Proceedings*.

[1] Presented at the Annual Meeting, New York, N. Y., January 15, 1936.

an important step toward the objective to which this Society has given so much earnest thought for more than twelve years—namely, to provide the Engineering Profession with authentic and readily accessible data pertaining to floods in American streams. Should further work of this class be undertaken by the Federal Government, it should properly receive the same hearty co-operation on the part of the Society that was extended during the past two years.

The opinion and recommendations of the Committee were requested by the Water Planning Committee of the National Resources Board as regards future flood-data compilations which might to advantage be undertaken with Federal funds. In substance, the Committee recommended that the National Resources Board should:

(1) Continue the tabulation of flood data to include all records of value, whether live or discontinued, which, either for lack of space or because of other considerations, it was necessary to omit from the report published by the U. S. Geological Survey in 1935;

(2) Make an inventory of destructive floods that have occurred in the United States, for all streams, including those on which gages have not been maintained;

(3) Make an inventory of cloudburst floods, their causes and frequency of occurrence, designating the areas in the United States chiefly affected and those rarely affected, or apparently immune;

(4) Promote research to discover the extent to which flood peaks in the larger rivers are being affected as the result of Man's occupancy of the land and to discover what reduction in the intensities and frequency of occurrence of flood peaks might be established by changing the storm run-off conditions in different parts of a drainage basin; and,

(5) Promote a much needed research in the economics of flood control.

Recommendation (1).—The additional records provided by Recommendation (1) should be published in substantially the form previously adopted, and should include the more important river-stage records obtained at Weather Bureau and Army Engineer gages at which discharges have not been determined. Special efforts should be made to list flood occurrences ante-dating the periods covered by continuous gage records, to the end that all floods of historic importance shall be taken into account. Field work will be required to locate high-water marks of early floods and to reference these to existing gages.

Recommendation (2).—The inventory involved in Recommendation (2) should be carried back as far as engineering and historical data may permit. It should be made as complete as practicable as regards the description of meteorological conditions which produced each flood. The Board was urged to discover the synchronism, if any, in the arrival of flood crests from the tributaries which produced the flood; the flood heights attained as noted on structures, landmarks, and gages, if any; make comparisons with heights of other notable floods; and assemble figures relating to flood damage. The

foregoing information should be compiled from published as well as unpublished accounts taken from official and private sources. Recommendation (2) was submitted to the National Water Planning Committee on March 15, 1935, in response to a request for suggestions for work which might be undertaken by unemployed engineers and other "white-collar" workers from funds disbursed by the Federal Emergency Relief Administration. The Committee was advised that the limitations in wage scales imposed by law made it impracticable to employ the class of individuals needed for work of the kind called for.

Recommendation (3).—Cloudbursts cause the greatest floods to which the smaller streams are subject, and, in the aggregate, cause more damage annually than is caused by floods in the larger streams. The subject merits attention. No systematic study of cloudburst floods has ever been undertaken.

Recommendation (4).—Essentially, a flood peak is the result of the synchronism with which individual flood crests descending from the tributaries chance to contribute to the advancing flood crest in the parent stream. The basic thought in Recommendation (4) is that flood peaks may be modified by breaking up the synchronism that has obtained in the past. There is indication that Man's occupancy of the valleys and his use of the higher lands have been responsible, in some cases, in accentuating the synchronism referred to, causing higher flood peaks, whereas, in other cases, it appears to be producing the opposite effect—de-synchronization, causing flood heights to be reduced. Expediting the storm run-off from one part of a drainage basin and retarding it from other parts of the same basin, if intelligently planned, may effectively destroy synchronism and decrease the intensity and frequency of floods. The methods to be utilized will vary with the nature of the soil, topography, vegetative cover, and climatic conditions of any given basin. Storm run-off may be accelerated by improving natural as well as artificial drainage courses, and by changing the vegetative cover to a rapid-draining type. Retardation of flood run-off, on the other hand, calls for covering large areas with grass crops and other slow-draining vegetation, by building reservoirs and ponds, and by diverting flood waters into channels designed to spread such waters over lands for absorption into ground-water storage. Considerable detailed field study is required to make this research work of value. Close observations during and immediately after storm periods, which have rarely been made in the past, are essential.

Recommendation (5).—There is no first-class treatise on the economics of flood control. The practice, widely used, of predicating the justifiable cost of flood-control works on the damage that has been inflicted by floods in the past, is open to serious question. There is no proof that flood damage and justifiable cost of flood control are related in any manner. The attitude of the Courts supports the view that only the benefits obtainable from flood control afford the proper criterion. Damage and benefits cannot be equated. Research, as recommended herein, will require full consideration of the historical aspects of the development of the flood menace and of the social, legal,

and economic aspects involved. It should define the true measures of benefits to be derived from flood control, and should produce suggestions as to methods for their evaluation.

Respectfully submitted:

GERARD H. MATTHES, *Chairman*,
FREDERICK H. FOWLER,
ROBERT E. HORTON,
IVAN E. HOUK,
CHARLES W. KUTZ,
CHARLES W. SHERMAN,
DANIEL C. WALSER,

December 23, 1935. Committee on Flood Protection Data.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

REPORTS

EQUITABLE ZONING AND ASSESSMENTS FOR CITY PLANNING PROJECTS[1]

PROGRESS REPORT OF COMMITTEE OF CITY PLANNING DIVISION

A review of the opinions of many officials and observers who have been actively connected with municipal finance and improvements, indicates the following principles to be operative in the field of special assessments:

(1) Legislative authority governs practice;

(2) Rigid restrictions in laws tend to prevent equitable assessments;

(3) The validity of an improvement proposal rests upon the need for it;

(4) Enhancement of value, being prospective, is difficult of determination;

(5) Equitable assessments depend largely upon qualifications of administrative officials; and,

(6) Administrative officials should hold office permanently and preferably should be attached to the assessor's office.

Principles (1) and (2)—Laws and Legal Restrictions.—The Investment Bankers Association of America has proposed a model law for special assessments.[2] In it were proposed (*a*) limits upon the amount of assessments both on individual properties and in the municipality as a whole; (*b*) a veto power by a majority of owners affected; (*c*) a prohibition in districts of high tax delinquency; (*d*) a hearing both before and after ordering improvements; (*e*) Court protection against inequities; and (*f*) full faith and credit of municipality pledged on bonds.

Mr. Philip A. Cornick[3] cites four types of special assessments: (1) Special charges for outlays made by a governing body for the abatement of a nuisance; (2) special charges for outlays resulting from the improvement of publicly owned appurtenances to private property; (3) special levies imposed on properties within a district created for the purpose, for all or part of the cost of specified public improvements constructed or services rendered within the district; and (4) special assessments imposed on properties benefited by public improvements or services in proportion to, and not in excess of, the amount of such benefit. Although Type (4) is the one commonly considered,

NOTE.—Discussion on this report will be closed in May, 1936, *Proceedings*.

[1] Presented at the meeting of the City Planning Division, New York, N. Y., January 15, 1936.

[2] 96 U. S., 97; 181 U. S., 324.

[3] "Municipal Finance", Chapter XII, Macmillan Co., N. Y., 1926.

principally on account of its obvious justice, actually it is rare. Inadequate or inefficient administrative machinery has made recourse to Type (3) necessary; the result is cost assessments instead of benefit assessments. The Supreme Court of the United States has sustained cost assessments without reference to benefits[2]; assessments may be made under the police power as well as under the taxing power. Equity and justice demand a relation as close as possible between costs and benefits. Competent administration is the best agency, but it must be free under the law to proceed as the conditions of each case indicate in the direction of justice and equity. Laws conceived without application to a wide variety of conditions may prevent even the most competent administrator from just procedure in distribution of assessment. "Courts presume that assessing officers act in accord with the law and the facts. The burden of proof is placed on him who contests the legality of an assessment to prove his case."[3]

Principle (3).—Demonstration of Need for Improvement.—Every considerable improvement has its place in a well considered plan of municipal economy and construction. A municipality without a comprehensive plan for expansion or for betterments without expansion is handicapped in all its projects. Without an efficient administrative machine it is likewise handicapped. The modern municipality and its environs to-day should serve the needs of all its population economically. Conflicting interests must be estimated and evaluated in these terms and plans developed on that basis; then each improvement of magnitude will, theoretically, occupy its proper time, place, and space; its priority and relative value can then be more clearly seen and determined. Thus, the aggregate program may at all times be kept within the financial resources of the city as a whole and the individual property owner protected from multiple assessments which place too concentrated a burden, from the standpoint of time, upon him. On the other hand, analysis of costs of going without certain improvements may be shown to be greater than the cost of the improvement, and it is such an analysis more than any other consideration which should be called upon to demonstrate a real need. This need should be demonstrated in advance of the initiation or proceedings. Demonstration of need consists in proving enhancement of the value of property equal to, or in excess of, the total cost of the improvement (except in the case of abatement of nuisance).

Principle (4).—Enhancement of Value.—In analyzing enhancement in property values through public improvements it appears to be desirable to consider different types of improvement separately. In each case it is assumed that the engineering design of the improvement is well conceived and well designed as to details (an assumption, unfortunately, which is not always true). Ordinarily, in any estimate involving physical values, the estimator has recourse to precedents. In the past, enhancement has been estimated with extreme variations of success. The error of estimate is reduced by an intimate knowledge on the part of an experienced assessor: Of land and property values, of land uses, of character and rate of population growth, of habits and change of habits, and of economic and commercial facts and tendencies.

In estimating the enhancement of value accruing to private property by the construction of specific public improvements, he has an alarming scarcity of records, and a low degree of precision is found in the available records. There appear to be almost no records of increasing value of property which separate the elements contributing to such increase. It would seem, therefore, that one of the necessary steps in any progress toward greater accuracy in estimating future enhancement of values of property due to public improvements would be the formulation of a program of investigation in each case where an improvement was proposed, which would include a record of values of property over a considerable period prior to the improvement, and a following record as to the course of values after the improvement. In such a study all other factors affecting valuation ought to be made, including the effect of the changing use of property, general and specific characteristics of population, and trends of habits and travel of population, trends of economic conditions, shifting of business and industrial centers, and, more specifically, the amount and character of traffic in front of and near the property; the nearness of transportation lines; the character of the neighborhood; and recent or contemplated improvements in the neighborhood. If such studies are not made, assessors must continue to guess at enhancement of values as they do to a large extent at present.

Principles (5) and (6).—Administrative Competency.—It is useless to expect that temporary boards of assessors or appraisers of property, the membership of which does not always consist of men of experience and judgment in estimating values, will make acceptable estimates. Such boards obviously cannot make the necessary studies. The only way to make progress toward more equitable assessments is to require the work of assessment to be done by men of suitable training who are also thoroughly familiar with local conditions and tendencies—in other words, men holding permanent positions, free from irrelevant influences. Such men should be attached to the city assessor's office where collateral information is available, and they should also have knowledge of the requirements of a comprehensive city plan as administered by a city planning board.

One writer[4] has stated that the criterion for any public improvement should be a demonstration of the need for it. The word, "need", may be subject to a number of interpretations. It is contended that if the improvement is an economic need, it will be paid for in one way or another whether the improvement is, or is not, made. It may also be contended that the need for a local improvement must be shown to bear some relation to the welfare of the entire community; in fact, the sporadic improvements generally made in the cities of the United States in earlier years without respect to their effect on the entire community, either present or future, have been recognized as inadequately conceived, and their consequences have contributed to the idea of city planning, which is now accepted generally and incorporated as part of the government of practically all large cities and many small cities.

It appears that the theory of special assessment is inadequate, and that it has at times been used to finance a proceeding that may itself be undesirable.

[4] "Municipal Finance", by A. E. Buck, Macmillan Co., N. Y., 1926.

Other situations involving the entire community or parts of the community outside the special benefit district might be cited. The extended ramifications of local public improvements, in many cases, cannot be comprehended in the absence of a well developed city plan and the continued presence in office of trained men thoroughly conversant with the theory and practical considerations upon which the city plan is based. Each city has its individuality and each city plan should reflect the city's development, correcting as far as possible the mistakes of the past, recognizing existing economic and social tendencies squarely and frankly, and projecting for the future of a city that which will best minister to the welfare of all the people.

Classifications of State Law on Special Assessments.—There are three broad bases for classifying a model State law on special assessments. Basis *A*, in outline form, may be stated as follows:

I.—Use of Local Improvements:
- (*A*) States limiting use to a marked degree.
- (*B*) States granting power for a few distinct purposes.
- (*C*) States setting out definitely a number of purposes.
- (*D*) States setting out definite purposes and granting sweeping power.

II.—Delegation of Power to Make Assessments:
- (*A*) General power in governing body without petition.
- (*B*) Power in governing body through petition of owners.
- (*C*) Power in voters or property owners by petition to governing body.
- (*D*) Power fully in voters or property owners.

III.—Share of Costs Borne by City:
- (*A*) Statute that sets definite percentage to be paid by city.
- (*B*) Statute that sets up requirement for city to pay costs of specific part of improvements, such as bridges and intersections.
- (*C*) Statute that grants power to Council to determine amount to be paid by city.
- (*D*) Statute that requires city to pay excess of benefits to private property.
- (*E*) Statute that requires city to pay for re-improvements.

IV.—Limitations on Assessments on Property:
- (*A*) States setting no limit but benefit to property.
- (*B*) States setting definite limit on assessments.

V.—Total Costs Placed on Abutting Owners or on Districts:
- (*A*) Total cost on abutting property.
- (*B*) Total cost on abutting property or districts.

VI.—Peculiarities of Certain States.

Basis *B* is a classification based on the character and limitations of indebtedness incurred for special assessments; and Basis *C* is a classification based on limitations on taxes that may be levied, and debt that may be incurred for general purposes.

A statistical analysis of answers to a questionnaire sent by the Committee to 153 cities gave unsatisfactory results as to classification, probably because of unknown variables, such as assessment procedures, tax and debt limitations, and possible inaccurate data. A summary of data on special assessments in cities of more than 30 000 population, as summarized by Mr. S. Graham, is given in Table 1. A bibliography of approximately 150 titles on the subject of special assessments has been compiled.

TABLE 1.—SPECIAL ASSESSMENTS IN CITIES OF MORE THAN 30 000 POPULATION

Year	Population	Property tax	Per capita	Special assessment	Percentage of property tax	Per capita
1908	24 065 539	$393 940 142	16.36	$44 020 405	11.17	1.83
1909	25 603 949	409 596 593	16.00	55 761 042	13.61	2.18
1910	27 316 407	485 084 672	17.76	64 723 589	13.34	2.37
1911	28 559 142	$425 065 780	14.90	$65 193 415	13.44	2.00
1912	29 320 579	512 289 648	17.51	68 401 758	13.35	2.34
1913	30 194 677	525 679 153	17.40	72 476 119	13.78	2.40
1914	....	....	......	....	....	......
1915	31 168 150	570 830 861	18.32	75 357 399	13.20	2.42
1916	32 267 415	$623 300 805	19.32	$68 835 819	11.04	2.13
1917	33 259 769	666 402 637	20.04	75 478 801	11.36	2.28
1918	34 326 669	705 723 158	20.52	68 058 351	9.64	1.98
1919	36 664 860	784 902 861	21.41	64 247 771	8.18	1.75
1920	....	....	......	....	....	......
1921	37 331 625	$832 817 234	22.30	$64 886 228	7.78	1.74
1922	38 736 657	1 337 784 348	34.51	103 134 949	7.70	2.66
1923	39 172 168	1 375 725 557	35.15	117 966 561	8.57	3.01
1924	39 981 105	1 491 234 371	37.34	143 661 129	9.63	3.59
1925	40 757 434	1 597 490 523	39.18	179 623 506	11.24	4.40
1926	41 840 033	$1 747 163 136	41.75	$199 747 779	11.43	4.77
1927	42 716 411	1 888 704 574	44.18	227 513 049	12.04	5.34

Discussion.—One member of this Committee, Mr. Shifrin, is of the opinion that under present governmental conditions, a competent continuing personnel cannot be obtained and, consequently, it is futile to attempt to secure a worthwhile approximation of the benefits to be derived by a large area included in a district some distance from the improvement. He suggests, therefore, that the City as a whole should pay for improvements other than those directly assessable to abutting property, the funds being derived from bond sales or special tax funds.

The Committee is agreed that no improvements should be undertaken in any community without first preparing a comprehensive city or regional plan and that all improvements undertaken thereafter must be a part of, and conform to, the adopted plan. One member, Mr. Cornick, is of the opinion that the "major problem in special assessments is not how to apportion the levy, but when and where levies of that type shall be prohibited entirely."

G. H. Herrold, M. Am. Soc. C. E., to whom the members of the Committee are indebted for assistance, expresses the opinion that:

"A special assessment is a bill for the cost of an improvement which is made by a governmental agency for its citizens. It should not be confused with the tax which is for the maintenance of governmental functions, and until this confusion of terms is straightened out, we leave the door open for

a lot of 'skulduddery.' A way should be found by which the property benefited should pay the cost, and this cannot be done by estimating what we think will be the increase in values. It must be a method which will work automatically to recoup the increase in value from the property benefited and have it apply to the cost of the project five, ten, fifteen, or twenty years after the work is done."

It appears to be obvious that, notwithstanding the diversity of opinion expressed concerning methods of apportionment of benefits, there is a general agreement that any equitable assessment must be preceded by a comprehensive and detailed city or regional plan and that projects executed should conform to that plan. Furthermore, benefits do not always accrue to land, but in many cases to individuals or corporations in the form of greater facility in recreation and transportation, and, consequently, are chargeable to larger groups of persons—in many cases to the entire community.

Acknowledgment is made to Mr. Herrold and to Mr. R. A. Dinkle for assistance in the preparation of this progress report.

Respectfully submitted,

FREDERIC BASS, *Chairman*,
PHILIP H. CORNICK,
H. SHIFRIN,
H. H. SMITH,
Committee of the City Planning Division on Equitable Zoning and Assessments for City Planning Projects.

December 31, 1935.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
REPORTS

PRINCIPLES TO CONTROL GOVERNMENTAL EXPENDITURES FOR PUBLIC WORKS[1]

FIRST PROGRESS REPORT OF COMMITTEE OF ENGINEERING-ECONOMICS AND FINANCE DIVISION

Seven broad principles to guide a Federal Public Works Program, may be defined as follows:

(1) The program should be based on sound economic consideration exclusively, and it should be formulated by a permanent and non-partisan Federal agency, preferably one of the character of the United States Supreme Court.

(2) Only those projects should be included that will answer a definite and well established public need which cannot be met reasonably by private capital.

(3) No project should be included which would duplicate, or impair the value of, existing property and facilities that afford adequate service to the public at reasonable rates.

(4) Projects should be classified in the order of their relative merit; this to be judged on the basis of national, rather than merely local, benefits.

(5) In classifying meritorious projects and in undertaking them, due weight should be given to their location with respect to the centers of unemployment, and to the labor they will require.

(6) No project which cannot be justified in the light of Principles (1) to (5) should be adopted merely to furnish employment.

(7) In evaluating the benefits which would result from a project, it is important that all items of cost of the project, both direct and indirect, be considered. This is particularly necessary when the project is to take the place of an existing facility.

Principle (1)

The program should be based on sound economic considerations exclusively, and it should be formulated by a permanent and non-partisan Federal agency, preferably one of the character of the United States Supreme Court.

Remarks.—Many works now under construction seem to have no justification. Without ignoring the obstacles in the way of such a tribunal, it is suggested that final authority in the formulation of the program should be confined to a body whose membership has the character and the permanency

Note.—Discussion on this report will be closed in May, 1936, *Proceedings*.

[1] Presented at the Annual Meeting, New York, N. Y., January 15, 1936.

of tenure of the United States Supreme Court. This might be called the United States Program Authority. This body should pass upon projects that have been reported as worthy of Federal support after the careful scrutiny by or of the bureaus of the Federal Government or, in the case of local projects, of corresponding local agencies. The procedure of investigation should require that every favorable report give definite facts to show that the favored project conforms with the principles herein laid down. It should also give some idea of the urgency of the need for the improvement.

Since changing conditions will introduce new considerations affecting the validity of former decisions, the "Program Authority" should review its program periodically and amend it from time to time. Such changes occur in the business and industrial world and it is equally true in the field of governmental activities. For example, ideas on transportation have been, or should be, completely changed by the development of the motor-bus, motor-truck, motor airship, motor-boat, Diesel-electric locomotives, and stream-lined railroad trains. The importance of highways and airways in the national transportation system has greatly increased and this may well prove to be true also in the case of waterways. It follows that a schedule of desirable public works that was drawn up some time ago might now need to be revised in order to give greater weight to these new developments.

PRINCIPLE (2)

Only those projects should be included that will answer a definite and well established public need which cannot be met reasonably by private capital.

Remarks.—The expression, "public need," used in Principle (2) requires definition. As the Committee uses the term, it covers things which any fairly large section of society can use with a resultant benefit in its economic or social status. As thus stated, the type of governmental organization under which the social group exists does not arise for consideration. It may be Communist, Fascist, or Capitalist, and, if the latter, action may be had either through the use of public funds or through private initiative, as may prove most advantageous. Under the capitalist system at any rate, since the amount of money available for public works is, or should be, quite definitely limited, projects should be given priority in accordance with their relative merits.

Projects may be divided into two general classes: (*A*) Those which deliver distinct services or are intended to produce specific economies (some of these projects result in revenues and others do not, but all of them afford some more or less exact basis for measuring benefits as compared with costs); and (*B*) those in which benefits are social and, therefore, general in character. Their benefits are incapable of being expressed in money values.

(*A*) If a project is for the supply of water, for a sewerage system, for the treatment of sewage and other wastes, for generating or distributing electric energy, or for both, for manufacturing or distibuting gas, or both, or for a public transportation system or highway, before it is adopted its economic

justification should be carefully investigated by an impartial and qualified agency as heretofore proposed. This agency should study the proposal; it should familiarize itself with the various plans that present themselves, evaluate them in accordance with their relative merits, and, having done so, hold public meetings at suitable places for the purpose of presenting to the interested citizens and their local officials full information concerning the plans under consideration. Every opportunity should be given for free and intelligent discussion of the benefits and the burdens likely to result from the project.

Utilities and utility services, such as the foregoing, are often desirable in that they add to the convenience of the citizens, to their general well-being, or to the satisfaction of their local pride. At the same time, their true and complete cost may be unduly high in comparison with their benefits, and it may often be found that the demand for the improvements is much the same as the urge to buy a new automobile when the existing car is far from having served its useful life. This test is more directly applicable to revenue-producing projects than to improvements, such as new highways, deeper river channels, and more commodious harbors where benefits are diffused, or are more or less debatable. It is assumed that even when a demonstrated need has been found to exist and it is in the class of those that have been, and can be, satisfactorily supplied by private capital, it will not be included in a Federal public works program.

(*B*) If a project is intended to confer a purely social benefit, it should be shown that this is a benefit that should be supplied at public expense and that it cannot be supplied in a less costly way. Among such projects are parks, school buildings, hospitals, asylums, city halls, and similar public buildings. Post office buildings to some extent also come within this description and so do grade crossings and improved housing.

In considering this class of projects, the proof of public need is a prerequisite but, even so, it should be shown that the project should be constructed at the expense of the entire public body and that there is no less costly way of meeting the need. For example, a new park might beautify a certain locality, erase a slum, add to the value of real estate, and conduce to the well-being of the people of a locality, and yet, the expenditure of money from the Federal Treasury, or even of funds contributed by all local taxpayers, may not be justified, because only a few will benefit, or because ample, accessible park areas already exist, or because equal and less costly benefits can be secured by enlarging existing parks, or by making access to them more convenient or more economical.

As to schools, similar questions arise. School authorities naturally wish the most modern buildings and equipment and have progressively added to their curricula with resultant demands for newer and larger buildings. Apparently, because of failure to scrutinize school budgets and school courses intelligently, new school buildings, even when really needed, have become unduly expensive. In some cases they have been built when, for less money, existing structures might have been improved adequately. In place of building new hospitals, it might be possible to make better use of existing ones

or, by enlarging those now available, to save overhead costs. New post offices and other public buildings ought to be built not to gratify local pride, but because they are really needed and because equal service can be had in no less expensive way.

The elimination of grade crossings may be a widespread benefit when the traffic is heavy; but there are so many crossings that the cost of their elimination should be carefully balanced against the resulting advantages. The apportionment of their cost may create difficulties that should be studied. Housing and slum clearance may have social values, but these are local. Such schemes may affect neighboring property unfavorably, and they always raise troublesome questions best handled by the localities themselves.

Principle (3)

No project should be included which would duplicate, or impair the value of, existing property and facilities that afford adequate service to the public at reasonable rates.

Remarks.—"Adequate" service is defined as service sufficient to supply the public demand, of the highest reasonable quality, and furnished at the lowest cost consistent with fairness to employees and with earning an equitable return on the legitimate investment in the property that serves the public. Principle (3) would exclude projects for reclaiming arid land or draining wet lands unless all other cultivable land in the country were being used to capacity, except possibly that, when the immediately tributary population happened to be in need of farm products for local consumption, it might be economically justifiable to reclaim or drain idle lands near-by. Even in this case, mature consideration should be given to supplementing the deficient local supply in other ways.

Furthermore, when a private building is rendered wholly or partly vacant by the construction of a publicly owned post-office building, the ability of the owner of private space, thus vacated, to pay taxes may be destroyed or impaired, his property may depreciate, and the local treasury may suffer. These possibilities should be carefully weighed.

Other projects excluded by Principle (3) are plants for manufacturing cement, fertilizer, or any other commodity now adequately provided; and electric power and gas plants unless for new markets which cannot be or are not served by existing agencies, private or public, at reasonable rates. The construction of new plants because a public official, high or low, asserts that the charges of the existing agencies, private or public, are too high, means their eventual destruction, the discharge of their employees, and loss not only of private but of public revenue. Certainly, in every such case there should be impartial and competent investigation before spending public funds in duplicating facilities for production.

A case in point is furnished by the recent controversy between the Tennessee Valley Authority and the manufacturers of cement. The Authority objected to the practice of the manufacturers in quoting a uniform price regardless of the point of origin of the cement. The best remedy did not

lie in the construction of a plant by the TVA, as was threatened, but in a proper effort to compel fair treatment by the manufacturers. This is better than the construction of a Government-owned plant which would probably destroy much more capital investment, and, consequently, tax revenue, than could be justified by the real savings to the TVA. Even the savings might be questioned since the operation of a new plant is certain to encounter difficulties. Moreover, proper accounting might wipe out the theoretical reduction of cost per barrel because in all likelihood the cost of the plant would have to be written off against a limited production.

It may be argued that this principle might serve to perpetuate an existing, uneconomical facility (as, for example, a power plant), by forbidding the construction of a new and more economical plant out of public funds. This objection may be valid, but in nearly every instance it ignores the fact that the total cost of the new plant to the entire social body must include the depreciated or replacement value of the old plant the usefulness of which is to be destroyed. The loss of the value of the old plant may not fall on the consumers; nor would the public body that constructs the new plant be asked to pay for the old one, but wealth will have been destroyed and to that extent the nation would be worse off.

Effects on the entire national wealth and income must be considered. Even though a new plant might be more economical, some method other than the destruction of the old plant, such, for example, as a fair proposal to write off some of the remaining investment in the old plant, seems the correct course. Such a compromise might not make jobs, but the aim in this case is to try to sort out worthwhile jobs and to eliminate anti-social ones.

The scheme of creating "yardsticks" for public utility plants is unnecessary, expensive, and unjustified. All the information that such "yardsticks" could possibly yield might be had from study of the numerous publicly owned electric, gas, and water supply plants that now exist. These plants are of all sizes and varieties and their capital and operating costs are either public records or may readily be obtained.

In considering additional or better highways and waterways, Principle (3) serves to supplement Principle (2). Frequently, proposed highways would be located parallel, not only to railways and waterways, but also to other improved highways. Similarly, proposed new or more capacious waterways may merely divert traffic from existing arteries of transportation. Eagerness to create a program of public works should not permit the inclusion of such items as these unless it is shown that there will be a net benefit to the country as a whole. In short, even demonstrably needed facilities should not be developed by the Government where existing facilities, whether publicly or privately owned, will, or can, be made adequate by existing ownership to serve probable demands on a reasonable basis.

Principle (4)

Projects should be classified in the order of their relative merit; this to be judged on the basis of national, rather than merely local, benefits.

Remarks.—The justification for Principle (4) is that the National Treasury will be called upon to bear all the cost of purely Federal projects and to underwrite all or part of the cost of any local projects in the program. It is appreciated that many problems will arise in applying Principle (4). In some cases benefits will be purely economic and, therefore, measurable in dollars. In other cases, economic advantages may be accompanied by less tangible ones, such as better sanitation, greater protection to life, or improved living conditions and general well-being. In still other cases, all benefits will be of the social character which cannot be evaluated in terms of money. Some weight must also attach to the employment afforded and its relation to general labor conditions. A project that is remote from large centers of population would possess less merit during emergencies that it might have during periods of normal business.

PRINCIPLE (5)

In classifying meritorious projects and in undertaking them, due weight should be given to their location with respect to the centers of unemployment and to the labor they will require.

Remarks.—A good method of treating this problem would be to classify projects on the basis of their ascertained merit, regardless of location and of their demand for labor, each accepted and approved project to be accompanied by an analysis of its requirements in labor and materials. Under normal conditions, each project would be taken up in the order of its merit, and as the program for any one year would include many projects, it is probable that they would be quite widely distributed. In an unemployment emergency, approved projects might be selected so as best to meet the then existing conditions.

PRINCIPLE (6)

No project which cannot be justified in the light of Principles (1) to (5) should be adopted merely to furnish employment.

Remarks.—A program which conforms with Principles (1) to (5) will include all the useful work that can be done. If it does not supply occupation for all the unemployed, the remainder should be cared for by relief in cash or in kind. This rule may not militate against "made" work not requiring material, provided it is worth while. Planners must recognize that there is a limit to the financial load that the nation can carry.

PRINCIPLE (7)

In evaluating the benefits which would result from a project, it is important that all items of cost of the project, both direct and indirect, be considered. This is particularly necessary when the project is to take the place of an existing facility.

Remarks.—The principal costs ordinarily encountered are itemized in Appendix H of the Report of Committee XXV (Waterways and Harbors) of

the American Railway Engineering Association.[2] These costs are clearly set forth in this Committee's report, as follows:

"The estimate of costs for a proposed project should include a careful and complete appraisal of all expenditures and charges pertaining to:

"1.—Initial Cost:

"(*a*) Construction cost, which should also include expenditures for any physical additions to, changes in, or adjustments of, existing highways, railways, waterways, or other property, whether publicly or privately owned, which will be necessitated by the proposed project.
"(*b*) Engineering and legal services from the inception of the project.
"(*c*) Accounting and administration expense.
"(*d*) Value of property taken or used.
"(*e*) Damage to property adversely affected.
"(*f*) Cost or value of all things or services contributed to the creation of the project by the owner, construction authority, or any other agency, for which direct and adequate compensation is not otherwise made.
"(*g*) Interest on all these items to the beginning of operation.

"2.—Development Cost:

"(*a*) For a reasonable initial period the excess of the annual charges over the direct revenues and tangible benefits accruing from the project.

"3.—Annual Costs:

"(*a*) Interest on initial and development costs.
"(*b*) Operation and maintenance of the project itself.
"(*c*) Increased costs of operation and maintenance of private and other governmental facilities and enterprises.
"(*d*) Accounting and administration.
"(*e*) Insurance.
"(*f*) Taxes that would be assessed against an equal and similar investment of private capital.
"(*g*) Depreciation including the element of obsolescence."

The appraisal of benefits which would result from the execution of any project should be a detailed one, each item being supported by a clear description of its nature and a statement of the evidence upon which the claim is based.

Respectfully submitted

CHARLES KELLER, *Chairman,*
W. W. DEBERARD,
FRED LAVIS,
L. C. SABIN,

Committee of the Engineering-Economics
and Finance Division on Principles
to Control Governmental Expenditures for Public Works.

May 9, 1935.

[2] *Bulletin 371*, Am. Ry. Eng. Assoc., November, 1934, pp. 238–239.

A

tov
the

fur
tio
pro
ap
suc
slo
an
me

me
hea
Al
fro
mu
as
Cr
ess

va
po
tal
of

Pro
193
and

p.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

ANALYSIS OF MULTIPLE ARCHES

Discussion

BY A. HRENNIKOFF, ESQ.

A. HRENNIKOFF,[26] ESQ. (by letter).[26a]—The favorable attitude of discussers toward the method presented in this paper, and their general agreement with the writer leave no necessity for an extensive closure.

To Professor Grinter and Mr. Newmark the writer extends his thanks for further elucidation of the principles underlying the method and of its relation to Professor Cross' method of moment distribution. Professor Grinter prefers solution of the multiple-arch system by Larson's method of successive approximations in which the joints are allowed to rotate and to translate in succession, with the adjacent joints kept fixed temporarily. However, a very slow convergence of the series which results from such a procedure presents an obstacle to the successful application of the method—a fact which is mentioned in the discussions by Professor Finlay and Mr. Newmark.

It is this objection that led Professor Cross to devise his neutral-point method of analyzing multiple arches, in which the movements of the pierhead are replaced by the movements of the neutral point of the joint. Although it also deals with successive approximations this method is free from the disadvantage of slow convergence, and presents a powerful tool in multiple-arch analysis. It is very original and ingenious, but not as simple as appears on the surface, and the brevity of its presentation by Professor Cross[19] requires a novice to do considerable thinking before grasping its essence thoroughly.

Undoubtedly, this feature has been a hindrance to assimilation of this valuable method by the profession, and the discussion by Professor Finlay, popularizing Professor Cross' analysis, is most timely and appropriate. He takes great pains to explain the neutral-point method and to improve the form of computations. The writer agrees with Professor Finlay that a proper

NOTE.—The paper by A. Hrennikoff, Esq., was published in December, 1934, *Proceedings*. Discussion on the paper has appeared in *Proceedings*, as follows: May, 1935, by Messrs. L. E. Grinter, N. M. Newmark, T. Y. Lin, A. H. Finlay, and A. W. Fischer; and September, 1935, by A. A. Eremin, Assoc. M. Am. Soc. C. E.

[26] Instr., Dept. of Civ. Eng., Univ. of British Columbia, Vancouver, B. C., Canada.

[26a] Received by the Secretary December 23, 1935.

[19] "Continuous Frames of Reinforced Concrete," by Messrs. Cross and Morgan, p. 316 *et seq.*

form, such as that of Fig. 18, is very necessary for orderly calculations of the results, especially if the method is intended for use in a designing office.

The same statement applies, of course, to the writer's tabulation of the solution for the four-span arch in Fig. 11. It is rather peculiar to notice in this connection that Professor Grinter does not approve of the table (Fig. 11), and believes that it obscures the physical side of the picture. This is exactly the opposite of the writer's opinion, and the suggested form of the table with small diagrams of arches on the left-hand side has been especially devised for the purpose of clarifying the successive physical steps taken. This tabular form, therefore, is an integral and important part of the method. It is practically self-explanatory if the reader only keeps in mind the following few simple facts:

1.—The given multiple arch is solved after considering arches of fewer spans;

2.—Each line contains moments and thrusts at various terminals in the structure drawn on the diagram when the encircled joint undergoes a rotation, α, or a horizontal displacement, Δ, of the magnitude stated in the column headed "Conditions";

3.—The necessary movements are determined by the solution of two simultaneous equations stated on the right-hand side and representing the conditions of equilibrium of the joint;

4.—The "joint distribution factors" utilized as coefficients before α and Δ in these equations are themselves determined along the same lines in the earlier part of the analysis (see the first ten lines of Fig. 11.).

The foregoing facts, as well as the supporting theory, seem quite simple to the writer; but of course he realizes that the advantage of familiarity may easily blind him to the difficulties inherent in his method. The judgment of disinterested engineers should be more reliable from this point of view; but most of the discussions are disappointingly general in nature, avoiding, for the most part, direct specific comments.

Mr. Fischer is the only discusser who made a detailed study of the writer's method as applied to a two-span arch. It is to be regretted that he did not extend his thorough investigation to the case of the four-span arch in which the advantages of the method become more pronounced. A two-span arch can be analyzed quite easily even by the otherwise cumbersome method developed by D. E. Larson, Jun. Am. Soc. C. E.

Mr. Lin's suggestion of using kip-inch units for moments instead of kip-feet units is not without merit. If inches and kips are used throughout for moments as well as for E, I, and A, the units of distribution factors cannot be in error. However, Mr. Lin is mistaken in claiming that such a choice of units provides a check on the solution. The equality of numerical values of certain distribution factors, discovered by him, follows from the identity of general expressions from which these numerical values are determined, and, consequently, does not check anything.

The problem of analyzing a multiple-arch system on elastic piers with flexible tie-rods in the arches, suggested by Mr. Eremin, suits the writer's

method admirably. The tie-rods should be treated as separate members. Columns should be added in the computation table at each joint for each tie-rod. The single-span, displacement, thrust factor for a rod can be expressed by the formula:

$$h = \pm \frac{AE}{l} \qquad (42)$$

the sign being plus for the far end and minus for the near end. The three other distribution factors for the rod are each equal to zero. The factors for the rods, of course, are included with the others in the calculation of the joint distribution factors. Introduction of tie-rods, therefore, does not increase the complexity of the problem, and causes only a slight increase in the numerical work. The solution outlined by Mr. Eremin, consisting in removing the restraints caused by tie-rods, and in equating the deformations of the arches to the changes in length of the tie-rods, is very long, and has the disadvantages of other algebraic solutions based on the so-called "first principles."

To summarize the preceding discussion, the following methods have been proposed to date for the solution of open-rib multiple arches on elastic piers: (1) The algebraic method involving a number of simultaneous equations for the determination of the movements at the pier-heads; (2) the Larson method; (3) the neutral-point method by Professor Cross; and (4) the writer's method.

Method (1) is generally a long process. Method (2) is quite satisfactory for two-span arches; but is unwieldly for a greater number of spans, except when the piers are very rigid. With more than two spans the choice should be between Methods (3) and (4), the data so far available being insufficient to enable one to make a final selection.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

ANALYSIS OF THICK ARCH DAMS, INCLUDING ABUTMENT YIELD

Discussion

BY PHILIP CRAVITZ, JUN. AM. SOC. C. E.

PHILIP CRAVITZ,[13] JUN. AM. SOC. C. E. (by letter).[13a]—In an engineering paper involving such ponderous equations as those concocted by the writer, it is indeed satisfying to him to obtain a verification of his basic equations by an independent method such as that provided by Mr. Nelidov. That the more laborious general equations of elasticity should preferably be used in the design of important structures, as is stated by Mr. Nelidov, is a debatable matter of opinion. It can scarcely be denied that an exact solution for an actual case in practice can probably never be obtained, because there are always such factors as the irregularity of the dam site, dissymmetry relative to any given center line, variations in the properties of the abutments when traversed from side to side, etc.

Because of such practical considerations, the writer believes that an exact theoretical solution may prove more bothersome, and scarcely more reliable, than a method which strives to consider all stress factors of equal magnitude of importance plus the judgment of an experienced design engineer. The curves presented in the paper do enable a designer to visualize quantitatively the effect of yielding abutments on stresses in thick arch dams.

However, because of the basic fact that the stresses of a statically indeterminate structure, such as an arch dam, are fundamentally independent of the scale, regardless of the number of stress determinants that are considered, there must be some mathematical set-up, similar to that devised by the writer, in which the final equations vary with $\frac{t}{r}$ and $2\,\phi_1$. Thus, it is only a question of regimenting computations so as to evolve curves that give stresses

NOTE.—The paper by Philip Cravitz, Jun. Am. Soc. C. E., was published in January, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1935, by Messrs. I. M. Nelidov, and A. Floris.

[13] Design Engr., Los Angeles County Flood Control Dist., Los Angeles, Calif.

[13a] Received by the Secretary December 14, 1935.

for any number of contributing factors introducing the slightest variation in the resultant stresses.

Regarding the desire expressed by Mr. Floris to obtain the curves in a universal system of units, it may be sufficient to indicate that the stresses given by the curves may be easily converted to any other desired system by the multiplication of a constant. For example, to obtain the stresses, in kilograms per square centimeter, multiply all values obtained from the curves by 0.23 times the height of water head, in meters.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

THE HYDRAULIC JUMP IN TERMS OF DYNAMIC SIMILARITY

Discussion

By Boris A. Bakhmeteff, M. Am. Soc. C. E., and Arthur E. Matzke, Jun. Am. Soc, C. E.

Boris A. Bakhmeteff,[54] M. Am. Soc. C. E., and Arthur E. Matzke,[55] Jun. Am. Soc. C. E. (by letter).[55a]—The many sided and helpful discussions prompted by the paper are gratifying. The endorsement of the manner of treatment by Professor Woods and the appreciative remarks with regard to its practical usefulness by Mr. Stevens are ample reward for the efforts of the writers to clarify a complex and, at times, puzzling field of study. Many new questions have been raised which cannot be answered adequately in the light of the knowledge available at present and which, therefore, usefully indicate subject matter for further research.

The writers are fully conscious of the limitations imposed on the results by the size of the flume. They agree entirely with the remarks by Messrs. Mavis, Luksch, Nelidov, and others regarding the possible effect of the channel dimensions on the friction and the longitudinal dimensions of the jump. In fact, final clarity on this subject can be obtained only through systematic and well-planned observations on a scale exceeding any of the experimental work heretofore cited. Furthermore, in comparing observations, it will be imperative in the future to adopt some unified basis in determining "the end of the jump." In fact, some of the discrepancies suggested by Fig. 13 and Fig. 14 may be partly due to differences in estimating where the jump terminates.

The uncertainty of all these features would seem to militate, at least for the time being, against any premature attempts to crystallize the avail-

Note.—The paper by Boris A. Bakhmeteff, M. Am. Soc. C. E., and Arthur E. Matzke, Jun. Am. Soc. C. E., was published in February, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1935, by Hunter Rouse, Esq.; May, 1935, by Messrs. Sherman W. Woodward, Robert E. Kennedy L. Standish Hall, and Morrough P. O'Brien; August, 1935, by Messrs. F. V. A. E. Engels, Baldwin W. Woods, and J. C. Stevens; September, 1935, by Messrs. Nolan Page, Andrei I. Ivanchenko, and F. T. Mavis and A. Luksch; and November, 1935, by I. M. Nelidov, Assoc. M. Am. Soc. C. E.

[54] New York, N. Y.

[55] Research Asst., Dept. of Civ. Eng., Columbia Univ., New York, N. Y.

[55a] Received by the Secretary December 28, 1935.

able knowledge in that, or in any other empirical, formula for computing the length of the jump (such as those suggested by Professor Ludin, and referred to in Mr. Kennedy's discussion; the formula proposed by Mr. Ivanchenko, Aravin, etc.).

The writers further appreciate the possible effect of "white water", as indicated by Messrs. Hall and Nelidov. This is also a question that can be answered only by special experiments. However, one should bear in mind that the basic formulas, Equations (8) to (17) do not carry, explicitly, the value of the specific weight, γ. Therefore, it would seem that presence of air in itself should not substantially affect the numerical relations in the jump, provided the average contents of air throughout the jump is not changed.

Any possible effect of the size of the streaming, to be considered eventually in practical engineering computations, will be limited to longitudinal elements only. As justly emphasized by Professor O'Brien, the vertical elements observed, even in the smallest flumes, are in such marvelous agreement with the theoretical curve, that scarcely any further "correction" is necessary. This fact would seem to justify the practical usefulness of Equations (8) and (16) and of dimensionless diagrams of the type of Figs. 3, 10, and 12, in their simplest form, without introducing any possible corrections, as mentioned by Professor O'Brien and Dr. Engel, to take into account the unevenness of the velocities and their actual distribution in the cross-sections. This correction, as established by Coriolis[56], requires multiplying the average velocity group in the momentum equation, $\frac{\gamma}{g}\,Q\,V = \frac{\gamma}{g}\,A\,V^2$, by a correction factor, $\alpha = \frac{\int v^2\,dA}{V^2 A}$; and in the energy equation the group, $\frac{\gamma}{g}\,Q\,\frac{V^2}{2} = \frac{\gamma}{g}\,A\,\frac{V^3}{2}$, by a factor, $\alpha' = \frac{\int v^3\,dA}{V^3 A}$, in which the respective integrals represent a summation over the entire area of the cross-section. The French hydraulicians of the Nineteenth Century gave much consideration to these points. Boussinesq in particular sanctioned the use of an average factor of about $\alpha' = 1.1$ and $\alpha = 1.03$. These numerical values were abstracted from observations on fully established, uniform flow patterns in canals and circular conduits. Now, flow in the section preceding the jump, where the kinetic energy is at its highest, usually follows a zone of intense acceleration (such as below a sluice or at the bottom of a spillway), the effect of which is to straighten out the velocities of the different filaments and to bring the velocity pattern in the lower-stage section before the jump close to practical uniformity. In the upper-stage section the influence of the kinetic energy is comparatively slight. However, to obtain some idea of the velocity picture in this region, special experiments were made in the Fluid Mechanics Laboratory of Columbia University, in New York City, the results of which are shown in Fig. 18. The velocity variance does not seem to be excessive.

[56] *Annales des Ponts et Chaussées*, 1836; see also, *Transactions*, A. S. M. E., Vol. 54, 1932, p. 57.

From the discussions by Messrs. Stevens and Page it would appear that dimensionless unified diagrams of the type of Figs. 12, 10, and 3, have become the order of the day. The value of such generalized diagrams lies exactly in eliminating the needless "series of mathematical gymnastics", so strongly objected to by Professor Woodward. There is no essential difference between the diagrams of Fig. 3 and those given by Mr. Stevens (Fig. 10) and Mr. Page (Fig. 12). Obviously, a relationship that exists between certain interdependent factors in a jump can be depicted equally well by selecting any of the factors that enter the problem as the independent variable against which all the other elements are plotted.

The choice of that or some other independent variable as a basis is largely a matter of taste and individual preference. The writers prefer λ, as in Fig. 3, or $d'_1 = \frac{d_1}{\epsilon_1}$[57], because the presentation in this case happens to be in direct and organic relation to the practical means of obtaining a jump as illustrated in Fig. 2. It is precisely for this reason that the curves in Fig. 3 indicate certain important physical features, which are entirely eclipsed in Fig. 10, or which can be derived from Fig. 12 only by indirect inference. The first is the course of d_j and its maximum value. Mr. Stevens takes exception to the notion itself, by stating: "There is no maximum height of jump * * *. The authors' maxima for d'_j and d'_2 in Fig. 3, are not physical characteristics * * * but merely consequences of the terminology * * *". This is a point of substantial importance. Obviously, the writers failed to make the matter sufficiently lucid. There is no better means of clarifying a point than by direct observation. Therefore, a special experiment was made using a layout similar to Fig. 2. The height, H, was kept constant and equal to 1.402 ft. The sluice-opening was changed, increasing the depth, d_1, and the discharge. By regulating the tail-water the jump was maintained in position near the sluice. With varying discharges, the lower and the upper stage depths (and thus the height of the jump, d_j) were measured. The data are assembled in Table 5.[58] As noted in Fig. 15 the height of the jump first increases until it

TABLE 5

No.	Q	d_1	d_2	d_j	V_1	λ	$\frac{d_2}{d_1}$
1	1.720	0.440	1.027	0.587	7.83	4.34	2.33
2	1.515	0.374	0.984	0.610	8.12	5.49	2.63
3	1.296	0.312	0.930	0.618	8.33	6.94	2.98
4	1.070	0.252	0.874	0.622	8.50	8.99	3.47
5	0.822	0.191	0.792	0.601	8.62	12.07	4.15
6	0.578	0.126	0.687	0.561	9.18	20.78	5.47

reaches a maximum value at $d_1 = 0.27$, after which d_j decreases. The value of $\frac{d_1}{H} = \frac{0.27}{1.4} = 0.193$ accords very closely with the theoretical value of $d'_1 = 0.206$ corresponding to $\lambda = 7.67$, at which the maximum is supposed to occur.

[57] "Hydraulics of Open Channels", by B. A. Bakhmeteff, M. Am. Soc. C. E., Eng. Soc. Monograph, p. 248.

[58] In Table 5, H = constant = 1 402 ft.

Mr. Stevens further questions the significance of the zone of undulating jumps which, as the writers claim, corresponds in Fig. 3 to the zone of the mounting d'_2-curve between $\lambda = 1$ and the maximum at $\lambda = 3$. The writers

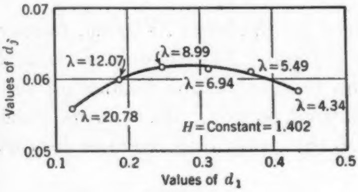


Fig. 15.

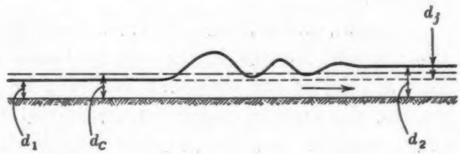


Fig. 16.

fully appreciate the potency of Mr. Stevens' remarks about the undulatory form of motion in the vicinity of the critical depth in general. However, they cannot escape the feeling that the undulatory jump is a separate and well determined phenomenon in itself. Strangely, it was the undular jump that was observed and described by most of the earlier investigators, for the simple reason that they worked with jumps that occurred in channels of steep slope, where flow in uniform motion was in a sub-critical (rapid) state. Compared to conditions at the foot of a dam or below a sluice the kineticity in uniform rapid flow is comparatively small, usually being far from sufficient to bring the jump into the direct roller form characterized by $\lambda \geq 3$ and $d'_1 \leq 0.4$. A schematic representation of an undulatory jump is given in Fig. 16. What differentiates these jumps from undulatory motions in the vicinity of the critical depth is the very substantial difference between the upper and lower stage, the depth ratio at times approaching the theoretical limit of $\frac{d_2}{d_1} = 2.$[59]

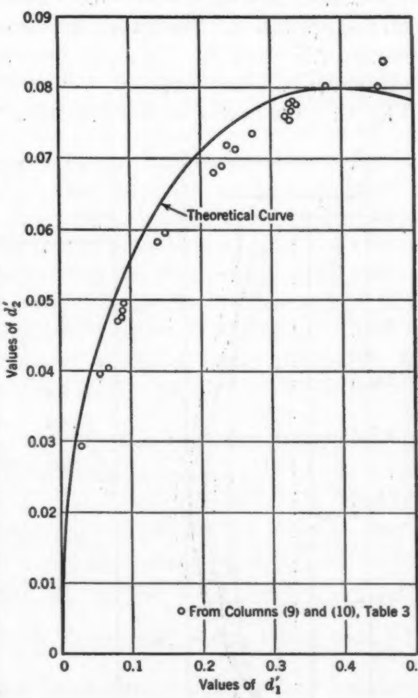


Fig. 17.

A distinct feature of the undular jump is the fact that, due to the curvature of the streaming, the momentum equation ceases to apply, and that observed values of d_2 are in excess of values computed from Equations (8) and (16).[60] This fact is confirmed once more by Mr. Page's observations.

---

[59] For detailed data see "Recherches Hydrauliques", par Darcy Bazin, Paris, 1865; also, "Berechnung der Wasserspiegellage", von Böss, V. D. I. *Forschungsarbeiten No. 284.*

[60] "Hydraulics of Open Channels", p. 251.

In fact, plotting the d'_2-values from Table 3 against d'_1, one arrives at Fig. 17. The points in the zone of direct jumps ($d'_1 < 0.4$) follow the theoretical curve rather satisfactorily, whereas the points for $d'_1 > 0.4$ fall outside the curve.

A point of particular interest is emphasized by Professor O'Brien, namely, the physical nature of the energy losses in a jump. The thought is that at least a part of the apparent loss may be due to a fallacious manner of estimating the kinetic energy in the initial and final section. As matters stand, these energies are computed on the basis of the prevailing average forward velocities $\left(\frac{V_1^2}{2g} \text{ and } \frac{V_2^2}{2g}\right)$, whereas it does stand to reason that at the upper stage, in addition to the average forward velocity, there might possibly be a substantial rotational effect. The senior writer is frank to admit that for a time he was disposed to attribute a considerable influence to this fact. He remembers having stated such views in informal discussions with members of the profession. Basically, the question reverts to the form of the velocity curve in Section d_2. To make the matter clear, at least qualitatively, special experiments were made at Columbia University which are illustrated by Fig. 18. By means of a Pitot tube, velocity diagrams were obtained in the

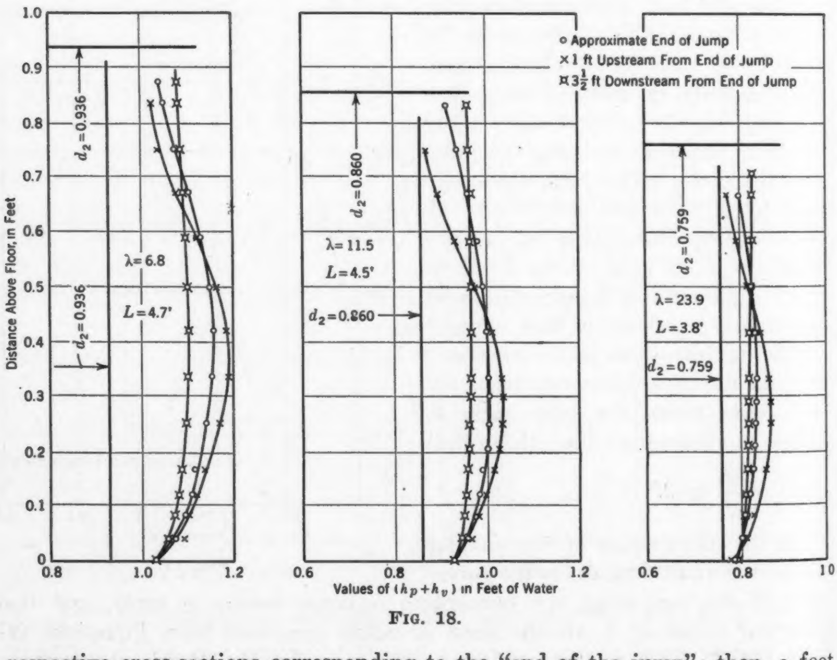


FIG. 18.

respective cross-sections corresponding to the "end of the jump"; then, a foot up stream nearer the roller, and, finally, down stream at a distance of $3\frac{1}{2}$ ft, where tranquil flow was unquestionably re-established. In the curves as given,

the abscissas represent the combined pressure and velocity heads registered by the Pitot tube and referred to the bottom of the flume. The pressure head, corresponding to hydrostatic distribution at the end section, with h_p assumed equal to d_2, is given by the d_2-vertical. Invariably, a certain relative rotation in an anti-clockwise direction is present in the "end-of-the-jump" section. This rotation is entirely eliminated $3\frac{1}{2}$ ft down stream. The rotation is relatively larger at small values of λ, and much less noticeable at the highest values of λ, where the loss of energy should be the greatest. Although these experiments are of a preliminary nature and must be substantiated by more detailed work, they would seem to testify against attributing any too great influence to relative rotation. It would seem that the correct explanation of the loss is that given in the discussion by Mr. Rouse. When expressing the kinetic energy in terms of the average forward velocity, one neglects the fact that a considerable portion of initial energy is absorbed by the kinetic energy of the additional turbulent agitation, which can be symbolized mathematically thus:

$$\Sigma \rho \frac{1}{2} [(u')^2 + (v')^2 + (w')^2]\ dx\ dy\ dz \ldots\ldots\ldots\ldots (56)$$

in which u', v', and w' are the local turbulent velocity components, variable in time and space, whereas the summation must be taken over the entire volume of the jump. As Mr. Rouse points out, the final molecular absorption of this energy occurs in the form of heat created by the additional viscous friction caused by excessive turbulent agitation. Obviously, such viscous friction is not limited to the average forward motion and is greatly intensified at every point by the presence of turbulent components complementary to the average motion. There is every reason to believe that the dissipation of this superfluous energy into heat is not consummated within the confines of the jump itself. In other words, the fluid leaves the end section of the jump still carrying a high content of excessive turbulence which is finally dissipated and brought down to the normal state of turbulent loss in established parallel flow only over a considerable distance.

A few more words of elucidation are required in connection with the remarks of Dr. Engel. There was no desire whatever on the part of the writers to "attack" the so-called "Boussinesq number" introduced by Dr. Engel in connection with his very valuable investigations of flow in Venturi flumes. The history of hydraulics shows that in the past many empirical parameters proved to be most useful. Consider, for example, the parametric basis, expressed in terms of $\left(a + \frac{b}{\sqrt{R}}\right)$, which Kutter and, later, Bazin used when building their empirical formulas for estimating open-channel roughness. Dealing with a complex problem of tri-dimensional flow, Dr. Engel may have struck a very fortunate chord in using a parameter of reference, to which the name "Boussinesq number" was given. What the writers believe, however, is that an empirical factor, no matter how practical and useful, should not be interpreted for a condition it cannot represent—namely, a mark of dynamic similarity. If the writers understand Dr. Engel correctly he believes

that, in view of the friction effect (which is disregarded in setting up the customary momentum equation for the jump, but which, obviously, is nevertheless present), jump data can be systematized more adequately by referring them to the Boussinesq number instead of using as a base the kinetic flow factor (or Froude number). In Dr. Engel's opinion this should become particularly evident in jumps where the depth at the foot of the jump exceeds the hydraulic mean radius.

To clarify this interesting question a special narrow channel, 2 in. wide, was built into the regular flume and a series of jumps was observed. The data are presented in Table 6.

TABLE 6.

Run No.	d_1	d_2	$\frac{d_2}{d_1}$	λ	R	$\frac{d_1}{R}$
N-1.........	0.222	0.764	3.44	9.31	0.0619	3.58
N-2.........	0.133	0.671	5.05	17.01	0.0522	2.55
N-3.........	0.078	0.551	7.06	29.38	0.0409	1.91
N-4.........	0.048	0.442	9.22	52.30	0.0308	1.56

As seen, the ratio of the initial depth to the hydraulic radius reaches $\frac{d_1}{R} = 3.58$; nevertheless, the experimental points in Fig. 19 do not deviate much from the theoretical curve. To clarify the matter further a still more direct experimental proof was sought. By appropriately varying the discharge and the initial depth, d_1, it was attempted to obtain a series of jumps at practically identical values of λ, but with substantially varying Boussinesq numbers. If the contention of Dr. Engel were correct the observed data referred to λ would scatter. On the other hand, when plotted against B (see Fig. 20), the points should manifest better regularity. The runs marked in Table 7

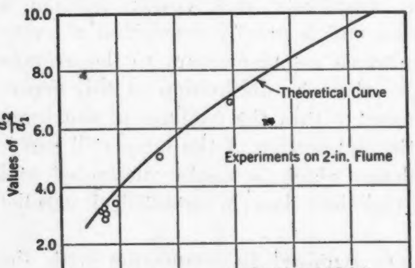


FIG. 19.

TABLE 7.

Run No.	d_1	d_2	V	r'	λ	B	$\frac{d_2}{d_1}$
C-1*A*......	0.300	0.896	8.76	0.135	7.93	4.19	2.99
C-1*B*......	0.242	0.744	7.84	0.128	7.89	3.88	3.07
C-1*C*......	0.162	0.521	6.32	0.113	7.65	3.27	3.22
C-1*D*......	0.107	0.344	5.10	0.096	7.56	2.90	3.21
N-1........	0.222	0.764	8.16	0.124	9.31	4.08	3.44
S-45.......	0.249	0.681	6.62	0.227	5.47	2.44	2.73
S-41.......	0.228	0.681	6.97	0.238	6.63	2.57	2.99
S-24.......	0.248	0.765	7.33	0.249	6.72	2.59	3.09
S-28.......	0.249	0.881	8.28	0.249	8.55	2.92	3.54
S-26.......	0.254	0.989	8.99	0.252	9.90	3.14	3.89
S-29.......	0.221	0.957	9.18	0.235	11.87	3.34	4.33

by the letter, *C*, were observed in the 2-in. flume with a value of λ actually varying between 7.93 and 7.56, or a variation of 5%, whereas the Boussinesq number varied about 36 per cent. Obviously, friction makes itself felt, but the observed ratios, $\frac{d_2}{d_1}$, form a compact group, with a variation of 7 per cent.

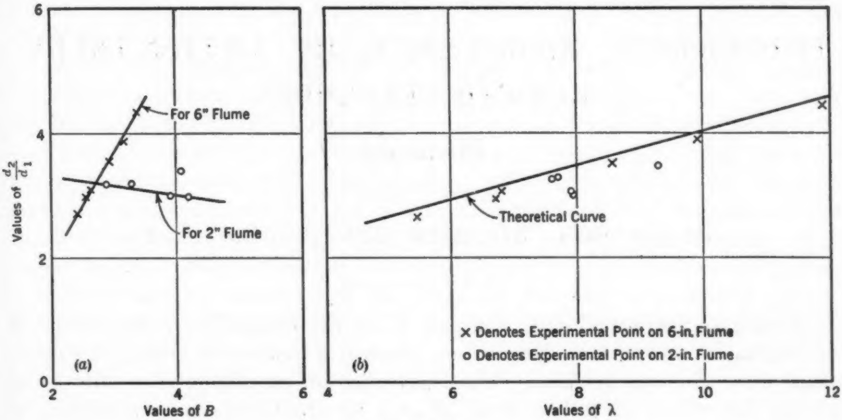


FIG. 20.

For further clarification Table 7 contains certain data, obtained at neighboring values of λ on the 6-in. flume, as given in Table 1. All the observed data when referred to λ, range quite consistently close to the theoretical curve in Fig. 20. By contrast, when referred to the Boussinesq number, the points for the 2-in. flume and the 6-in. flume form two separate and incompatible curves.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

FRICTIONAL RESISTANCE IN ARTIFICIALLY ROUGHENED PIPES

Discussion

BY VICTOR L. STREETER, JUN. AM. SOC. C. E.

VICTOR L. STREETER,[27] JUN. AM. SOC. C. E. (by letter).[27a]—In his analytical discussion of manometers, Mr. Wilson presents a method of finding the maximum error for one reading. The scattering of experimental results (Fig. 11(*a*)) for values of **R** less than 10^5 can be attributed, unquestionably, to inaccuracies of the manometers. Results and conclusions drawn from the paper were based on friction factors for Reynolds numbers of 10^6. For values of **R** equal to 10^6 and greater, losses of head in all cases, using rough pipes, were greater than 17.0 ft. As stated by Mr. Wilson, the maximum error for one reading of the manometer is less than 1% for pressure differences greater than 1.65 ft of water. The manometer with a solution of specific gravity equal to 1.05 was used for only a few readings. Its action was so sluggish that it would frequently require 30 min to reach equilibrium.

Recently, the writer has been using the micro-manometers described by Nikuradse[9], which are constructed in such a way that air-water or water-mercury may be used as fluids. This arrangement provides for a large range of pressure differences and a high degree of accuracy.

Dr. Folsom's investigation clearly shows the effect of spacing of roughness elements on turbulence formation. A new work by H. Schlichting[28], based on experiments made at the Kaiser Wilhelm Institute for Fluid Research, at Göttingen, presents a simplified method of obtaining friction factors for practically any type of reproducible roughness. The results show that the maximum friction factor is obtained with comparatively small density of roughness elements. In his paper, Schlichting also uses Equation (10) as a basis for comparing various roughnesses.

NOTE.—The paper by Victor L. Streeter, Jun. Am. Soc. C. E., was published in February, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: August, 1935, by Messrs. Warren E. Wilson, Richard G. Folsom, and Ralph W. Powell.

[27] Freeman Scholar, Am. Soc. of Mech. Engrs., Göttingen, Germany.

[27a] Received by the Secretary January 11, 1936.

[9] "Gesetzmässigkeiten der turbulenten Strömung in glatten Röhren". V. D. I., *Forschungsheft* No. 356, 1932.

[28] *Ingenieur Archiv*, 1936.

In the roughness utilized by the writer a pitch of 11.5 to the inch was used. In a 2-in. pipe the angle between the groove and a plane perpendicular to the axis of the pipe is 0° 48′. This slight divergence from circular grooves perpendicular to the axis of the pipe undoubtedly has some tendency to increase the friction factor. The writer is of the opinion, however, that the differences in friction factors in Table 4 result from differences in the shape of roughness elements, which appears to have as much effect as their size.

In Fig. 17, Professor Powell presents an interesting study of friction factor-Reynolds number relationships. Equation (15) agrees very well with the Blasius Equation (6). The friction factors for Reynolds lead pipes, however, are surprisingly low, resulting in values as much as 25% less than for the brass pipes used by Saph and Schoder. This great variation for "smooth" pipes was not confirmed by Drew, Koo, and McAdams[10], who determined that the friction factor for smooth pipes would not vary more than ± 5% from Equation (8). This conclusion was based on a study of 1 400 observations made by several investigators on various kinds and sizes of smooth pipes.

[10] "The Friction Factor for Clean Round Pipes", *Journal*, Am. Inst. of Chemical Engrs., Vol. 28, p. 56, 1932.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

STRUCTURAL BEAMS IN TORSION

Discussion

BY INGE LYSE, M. AM. SOC. C. E., AND BRUCE G. JOHNSTON, JUN. AM. SOC. C. E.

INGE LYSE,[34] M. AM. SOC. C. E., AND BRUCE G. JOHNSTON,[35] JUN. AM. SOC. C. E. (by letter).[35a]—The original analysis by Professor Westergaard and Mr. Mindlin with regard to stress concentration in fillets is a most worthy contribution on this subject, which was subsequently expanded and studied in the light of experimental tests with soap films on both I-beams and angle sections by Mr. Ehasz, and so reported in his discussion. Mr. Ehasz found that their analysis applied well for the stress concentration in I-beams. Professor Reynolds gave close attention to the investigation and aided liberally in the analytical studies presented in the paper. His discussion of the more general study of I-beams subjected to torsion is a worthy addition to the typical cases considered by the writers. Mr. Werner has furnished additional bibliography on the subject of torsion and has contributed general formulas similar to those given by Professor Reynolds, for the case of a beam either fixed or free at the ends and twisted by a couple at any point along the beam.

The writers still believe that the use of higher unit stresses for the direct longitudinal stresses at the extreme corners of the flanges of beams at their fixed ends is justifiable. All the fixed-end tests, as originally reported, show that at the torsional yield point of the beam both the theoretical and measured stresses at the ends of the beam were roughly 50% higher than the yield-point strength of the material in direct tension.

In connection with the more practical problems of combined bending and torsion, with the use of ordinary fabrication processes and present assumptions accepted in building design, the outer ends of beams at points where they are riveted to a column or girder will probably be considered unrestrained. Even

NOTE.—The paper by Inge Lyse, M. Am. Soc. C. E., and Bruce G. Johnston, Jun. Am. Soc. C. E., was published in April, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: April, 1935, by Messrs. H. M. Westergaard and R. D. Mindlin, and Joseph B. Reynolds; August, 1935, by Messrs. Harold E. Wessman, and F. B. Seely and W. J. Putnam; and October, 1935, by Messrs. P. Wilhelm Werner, F. L. Ehasz, W. P. Roop and John B. Letherbury, and Alfred T. Waidelich.

[34] Research Associate Prof. of Eng. Materials, Lehigh Univ., Bethlehem, Pa.

[35] Instructor in Civ. Eng., Columbia Univ., New York N. Y.

[35a] Received by the Secretary January 15, 1936.

in a heavily built-up welded end connection it would not be possible to rely upon 100% fixity. At some section along the beam, however, a plane of torsional fixity is maintained, but the accompanying direct stresses are localized in the outer edges of the flanges. Since each flange acts as a separate rectangular beam section, its reserve strength after plastic action commences is much greater than in the usual case of an I-beam in bending. Another factor, which does not lend itself readily to mathematical calculation, is the restraining action of walls, brickwork, intermediate floor-beams and the "giving" of the columns or girders into which the beam in combined bending and torsion is framed.

The questions raised by Professor Wessman have been answered in detail in the discussion by Mr. Ehasz and in a later descriptive paper.[36] The record of experimental data on the torsional properties of channel section as furnished by Professors Seely and Putnam is a valuable addition to the writers' experiments on I and H-sections. Mr. Ehasz's continuation of the writers' soap film experiments shows that this method serves well for the study of the concentration of torsional stress in structural sections as well as for the evaluation of the torsional rigidity. The remarkable uniformity in results obtained by Mr. Ehasz gives confidence in the experimental work and indicates how successfully the membrane analogy may be applied to analytical problems. The data given in Fig. 39 and Fig. 40 might have been presented also in a more general form with the known accurate formula (Equation (6)) for rectangles as a basis. The additional torsional rigidity due to the angular shape would then be represented as a definite function of fillet radius and the thickness of the legs of the angle.

The writers are interested in the results reported by Messrs. Roop and Letherbury on closed sections. Unfortunately, the writers did not investigate such sections and, therefore, are unable to account for the apparent loss in torsional rigidity shown in their tests. However, since the same effect was noted in the case of the ring, the torsional properties of which may readily be analyzed precisely and which have been corroborated by numerous tests, it is possible that the welded seam did not provide complete continuity of material.

Professor Waidelich has raised questions which should rightly be the subject of further study. In conclusion, the writers are very grateful for the general interest evidenced in the paper by all the discussers. It is particularly gratifying to record the interest in more exact methods of analysis of structural members shown by the steel industry.

[36] "Torsional Rigidity of Structural Sections", by Bruce G. Johnston, Jun. Am. Soc. C. E., *Civil Engineering*, November, 1935, pp. 698–701.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

ADAPTATION OF VENTURI FLUMES TO FLOW MEASUREMENTS IN CONDUITS

Discussion

BY MESSRS. F. V. A. E. ENGEL, AND J. C. STEVENS

DR. ING. F. V. A. E. ENGEL[17] (by letter).[17a]—In comparison with other measuring devices, such as weirs, for example, the investigation of the Venturi flume meter extends over only a short period of about twenty years. This paper, therefore, is welcome as a further contribution on the subject. It certainly merits the careful study of hydraulic engineers who are confronted by problems relating to the measurement of large quantities of fluid flow.

The writer has been engaged for several years in investigating and designing Venturi flume meters and would like to offer the following comments. In connection with Equation (14), for a free discharging Venturi flume with a rectangular throat, reference should have been made to the investigations by Mr. A. H. Jameson. In 1925, Jameson[18] published equations relating to throat sections of different shapes; that is, a rectangle, triangle, parabola, and a trapezium. Definite progress was made by Jameson[19] in a paper published in 1930, but this work does not seem to have become generally known. The equations developed make it possible to determine the dimensions of a Venturi flume of rectangular section by a direct method, which seems to be much simpler than to solve Equation (14) graphically. Particularly in the case of a rectangular section of both the throat of the flume and the channel, the Jameson method for obtaining the various dimensions is so straightforward that it is the only one to be recommended.

Possibly the accuracy obtained by the authors with their method of calibration would have been greater had they been able to conduct the tests on the Venturi flume in a hydraulic laboratory. The results they obtained by

NOTE.—The paper by Harold K. Palmer, M. Am. Soc. C. E., and Fred D. Bowlus, Assoc. M. Am. Soc. C. E., was published in September, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1935, by Messrs. N. F. Hopkins, and Hunter Rouse.

[17] Wembley, Middlesex, England.

[17a] Received by the Secretary December 10, 1935.

[18] "The Venturi Flume and the Effect of Contractions in Open Channels", by A. H. Jameson, *Transactions*, Inst. of Water Engrs., Vol. 30 (1925).

[19] "The Development of the Venturi Flume", by A. H. Jameson, *Water and Water Engineering*, Vol. 32 (1930), pp. 105 to 107.

comparing their flume with the Parshall meter are not very convincing, as an accuracy of the order of 5% would appear rather unsatisfactory.

To the writer's knowledge, the Parshall meter has not been tested in a hydraulic laboratory and, therefore, its discharge coefficient is not known with sufficient accuracy. The discharge coefficients of the Parshall meter and the design by Messrs. Palmer and Bowlus may have quite different characteristics and it seems a rather doubtful method, therefore, to calibrate one Venturi flume meter against another. The only reliable method of calibration would appear to be either by weighing or by the determination of water-volume passing through the flume in a measured time.

A further point which does not seem very satisfactory is that the calibration only covered a range of Q-values equal to about 1 to 5, whereas a range of about 1 to 8 or 1 to 10 is usually demanded.

TABLE 6.—DISCHARGE COEFFICIENTS, VENTURI FLUME; THROAT LENGTH, 20 INCHES

Test No.	Rate of flow, Q, in liters per sec*	DEPTH OF FLOW, IN CENTIMETERS †		WIDTH, IN CENTIMETERS †		Discharge coefficient C_f	Deviation from the mean value of C_f (percentages)	Discharge coefficient, C'_f	CONSTANTS, RELATED TO CHANNEL SECTION UP STREAM FROM FLUME	
		Up stream, d_2	Throat, d_3	Up stream, b_2	Throat, b_3				Boussinesq number	Froude number
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
				(a) FLUME NO. 4, 1932; WIDTH RATIO, $\frac{b^3}{b_2}=0.294$						
1	37.171	39.44	31.27	31.30	9.17	0.9433	−0.52	0.9676	0.203	0.153
2	30.764	34.65	27.19	13.30	9.18	0.9468	−0.15	0.9638	0.194	0.154
3	24.205	29.49	22.91	31.30	9.19	0.9482	0	0.9648	0.185	0.154
9	24.208	29.49	22.97	31.30	9.19	0.9483	+0.01	0.9648	0.185	0.154
10	24.211	29.50	22.92	31.30	9.19	0.9481	−0.01	0.9645	0.185	0.154
33	24.204	29.62	23.07	31.30	9.14	0.9465	−0.18	0.9634	0.184	0.153
11	18.430	24.60	18.83	31.30	9.19	0.9472	−0.11	0.9640	0.175	0.154
12	13.365	19.76	14.78	31.30	9.21	0.9516	+0.36	0.9690	0.165	0.155
19	13.344	19.77	14.81	31.30	9.21	0.9495	+0.14	0.9668	0.164	0.155
20	13.344	19.77	14.79	31.30	9.21	0.9495	+0.14	0.9668	0.164	0.155
41	13.232	19.69	14.77	31.25	9.17	0.9512	+0.32	0.9687	0.164	0.155
40	13.111	19.57	14.69	31.25	9.17	0.9517	+0.37	0.9686	0.164	0.155
21	8.446	14.58	10.58	31.30	9.26	0.9445	−0.39	0.9611	0.152	0.155
22	4.471	9.49	6.67	31.05	9.28	0.9486	+0.04	0.9665	0.141	0.157
				(b) FLUME NO. 3, 1932; WIDTH RATIO, $\frac{b^3}{b_2}=0.533$						
22	51.897	31.96	25.18	31.20	16.6	0.9526	−0.50	1.0149	0.363	0.294
23	49.538	31.12	24.45	31.20	16.6	0.9469	−1.10	1.0081	0.358	0.293
1	46.88	29.82	23.32	31.20	16.6	0.9547	−0.28	1.0171	0.356	0.294
3	46.88	29.82	23.30	31.20	16.6	0.9547	−0.28	1.0171	0.356	0.294
7	46.88	29.80	23.30	31.20	16.6	0.9554	−0.21	1.0182	0.356	0.294
40	46.294	29.66	23.10	31.20	16.6	0.9598	+0.25	1.0128	0.354	0.294
24	39.885	26.78	20.61	31.20	16.6	0.9542	−0.33	1.0169	0.344	0.294
59	35.369	24.64	18.91	31.20	16.6	0.9553	−0.22	1.0188	0.336	0.295
25	30.892	22.53	17.05	31.20	16.6	0.9575	+0.01	1.0208	0.327	0.296
8	25.253	19.61	14.59	31.10	16.6	0.9619	+0.47	1.0276	0.318	0.298
11	25.181	19.63	14.63	31.10	16.6	0.9585	+0.12	1.0231	0.317	0.297
12	25.241	19.64	14.63	31.10	16.6	0.9601	+0.28	1.0248	0.318	0.298
13	25.301	19.64	14.64	31.10	16.6	0.9616	+0.44	1.0273	0.318	0.298
48	25.225	19.67	14.70	31.10	16.6	0.9574	0	1.0219	0.317	0.297
26	19.887	16.70	12.22	31.10	16.6	0.9640	+0.69	1.0295	0.305	0.300
63	16.407	14.67	10.65	31.10	16.6	0.9657	+0.87	1.0318	0.296	0.300
27	13.436	12.86	9.25	31.10	16.6	0.9633	+0.62	1.0288	0.286	0.299
53	8.824	9.74	7.28	31.10	16.6	0.9613	+0.41	1.0256	0.269	0.298
54	8.824	9.74	7.24	31.10	16.6	0.9613	+0.41	1.0256	0.269	0.298
16	8.732	9.69	7.15	31.00	16.6	0.9584	+0.10	1.0228	0.270	0.298
17	8.732	9.69	7.15	31.00	16.6	0.9584	+0.10	1.0228	0.270	0.298
21	8.725	9.71	7.17	31.00	16.6	0.9549	−0.26	1.0189	0.266	0.297
28	7.365	8.68	6.43	31.00	16.6	0.9542	−0.33	1.0176	0.262	0.296
29	6.140	7.74	5.72	31.00	16.6	0.9462	−1.17	1.0075	0.255	0.294

* 100 liters per sec = 3.54 cu ft per sec. † 1 cm = 0.3937 in.

It is the opinion of the writer that the Venturi flume meter, if correctly designed, is one of the most accurate of measuring devices, and that it has great advantage in comparison with the weir method.

Table 6 contains some test results on Venturi flume meters obtained during previous investigations[20] in the hydraulic laboratory of the City and Guilds (Engineering) College, in London, England, in 1932. Both the Venturi flumes were installed in a rectangular channel and had a rectangular throat section. The discharge equations used were:

$$Q = \left(\frac{2}{3}\right) C_f \sqrt{g} \ \ b_3 \left(d_2 + \frac{V_2^2}{2g}\right)^{\frac{3}{2}} \ldots\ldots\ldots\ldots (46)$$

and,

$$Q = \left(\frac{2}{3}\right)^{\frac{3}{2}} C'_f \sqrt{g} \ \ b_3 d_2^{\frac{3}{2}} \ldots\ldots\ldots\ldots\ldots\ldots (47)$$

in which C'_f is the product of C_f and the velocity-of-approach factor. Further data may be obtained from the previous publications by the writer[21] on the subject. For the flume with a width ratio of 0.294 (Table 6(*a*)), the average coefficient of discharge is 0.9482 (see Column (6)). The rate of flow was varied over a measuring range of about 1 to 8. It is of interest to note that there is only one test for which the discharge coefficient is more than 0.5% from the average. All the other test results are within the limits of ± 0.5 per cent. For a width ratio of 0.533, the test results of which are given in Table 6(*b*), there are only two test points outside the limit of ± 1%, whereas 19 of the total of 24 were within the limits of ± 0.5%, the average coefficient being 0.9574. Seldom under actual working conditions could an accuracy like this be obtained, but one can assume that the accuracy of a Venturi flume meter in a rectangular channel and using dimensions similar to those used in the writer's investigations, would be in the order of ± 1.5 per cent.

The section entitled "Energy Losses", seems rather misleading. The problem treated only refers to the dissipation of energy in the convergent entrance section and the authors' conclusion is that there is practically no dissipation of energy—a point which is evident from previous research. The more important point regarding the head losses due to the Venturi flume was totally neglected by the authors, whereas this is of the greatest practical value.

In designing Venturi flume meters, it is absolutely necessary to ascertain whether free discharge conditions prevail for certain down-stream conditions. There is a minimum head loss for which the flume will discharge without banking up the up-stream level for a given rate of flow. For a Venturi flume with a width ratio of 0.533, the writer found that in a channel with a horizontal bottom, it was possible to increase the down-stream depth without changing the state of free discharge, to a level corresponding to 92% of the up-stream depth. In a flume of a width ratio of 0.294, however, the down-

[20] "Non-Uniform Flow of Water: Problems and Phenomena in Open Channels with Side Contractions", by F. V. A. Engel, *The Engineer*, Vol. 155 (1933), pp. 392 to 394, 429–430, and 456–457.

[21] "The Venturi Flume", by F. V. A. E. Engel, *The Engineer*, Vol. 158 (1934), pp. 104 to 107 and 131 to 133.

stream depth was between 74% and 90% of the up-stream depth for the limit of free discharge conditions.

In order to determine head losses, the writer has developed a certain criterion based on the Boussinesq number[21, 22]. The critical Boussinesq number determines the head loss, which may be about 8%, or a value between 10 and 30 per cent. The critical Boussinesq number for the second flume is 0.358, and from this it will be seen that only for Test No. 22 (Table 6(*b*)) will the head loss be greater than 8 per cent. In the case of the flume with the narrow throat, the critical Boussinesq number is 0.147 and from Column (9), Table 6(*a*), it is evident that in all cases, except the last line, the head losses will be greater than 8 per cent.

The critical Froude number determines whether the flume is discharging freely, and the test results are quite in accordance with the critical numbers obtained by calculation. The critical Froude number for the flume with the width ratio of 0.294 is 0.154, referred to the up-stream section, whereas for the second flume, the critical Froude number is 0.296. The last columns of Table 6 show also a characterizing feature of both the Froude and Boussinesq numbers, namely a constant value of the Froude number and a comparatively large variation in the Boussinesq number.

J. C. STEVENS,[23] M. AM. SOC. C. E. (by letter).[23a]—A very practical application of the use of critical flow for measuring the discharge in conduits is demonstrated in this paper. If critical flow can be produced in a conduit by any kind of constriction whatever, the depth above that constriction immediately becomes a stable index of discharge.

Caution must be observed, however, in designing the control so that it is not submerged for the range of discharges that may obtain. Whenever the tail-water level is more than about 65% of that of the head-water over the crest of the control, the head-water rises with the tail-water and loses its stability as an index of flow. Two depths must then be observed and discharge determination from them becomes more complex and uncertain. Recorders with totalizing equipment also lose their efficacy at this point of submergence.

In circular conduits the writer has used a "hump" control on the bed of the conduit to produce critical flow over it. The flow is readily computed from the criterion that for critical flow the velocity head equals one-half the mean depth. This applies to any shape of conduit whatever as long as width is a continuous function of the depth.

Fig. 18 shows the type of control used in a circular conduit, which is believed to have some advantages over the trapezoidal control used by the authors and certainly involves less energy losses. It is readily constructed of concrete without forms; it may even be pre-cast and placed while the conduit is in service. Fig. 18 also shows the rating curves for five heights of the con-

[22] "Venturi-Kanalmesser: Die messtechnischen Eigenschaften in Abhaengigkeit von den Stroemungsarten", von F. V. A. E. Engel, *Archiv für Technisches Messen*, V. 1253–2, March, 1935, No. 45.

[23] Cons. Hydr. Engr. (Stevens & Koon), Portland, Ore.

[23a] Received by the Secretary December 13, 1935.

trol section. Discharges are given for depths in the conduit just above the control and both depths and discharges are in terms of the inside diameter of the pipe.

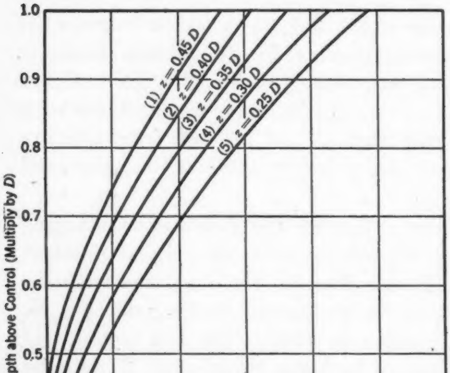


FIG. 18.

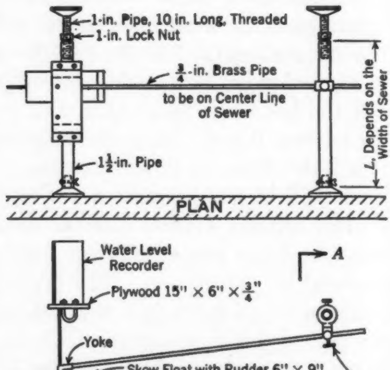


FIG. 19.

Table 7 is a sample of the computations for one of the curves, assuming D equal to unity. To obtain corresponding values for any other diameter, the tabular values are multiplied by the function of the diameter indicated at

TABLE 7.—SAMPLE COMPUTATIONS FOR CURVE 4, FIG. 18

$$\left(\frac{z}{D} = 0.30;\ \text{AND}\ \frac{Z}{D^2} = 0.198\right)$$

$\frac{y_c}{D}$ (1)	$\frac{z+y_c}{D}$ (2)	$\frac{X}{D}$ (3)	$\frac{X}{D^2}$ (4)	$\frac{0.785-(Z+X)}{D^2} = \frac{Y_c}{D^2}$ (5)	$\frac{t}{D}$ (6)	$\frac{Y_c}{t} \div D = \frac{y_m}{D}$ (7)	$\frac{1}{2} y_m \div D = \frac{h_2}{D}$ (8)	$\frac{\sqrt{2 g h_c}}{\sqrt{D}} = \frac{V_c}{\sqrt{D}}$ (9)	$\frac{Y_c V_c}{D^{3/2}} = \frac{Q}{D^{3/2}}$ (10)	$\frac{y_c + h_c + Z}{D} = \frac{e}{D}$ (11)	$\frac{A^*}{D^2}$ (12)	$\frac{Q}{A} \div \sqrt{D} = \frac{V_1}{\sqrt{D}}$ (13)	$\frac{V_1^2}{2g} \div D = \frac{h_1}{D}$ (14)	$\frac{e_h - h_1}{D} = \frac{y_1}{D}$ (15)
0	0.3	0.7	0.587	0	0.916	0	0	0	0	0.300	0.198	0	0	0.30
0.1	0.4	0.6	0.492	0.095	0.980	0.097	0.098	1.77	0.168	0.448	0.341	0.49	0.004	0.44
0.2	0.5	0.5	0.393	0.194	1.000	0.194	0.097	2.50	0.485	0.597	0.489	1.01	0.016	0.58
0.3	0.6	0.4	0.293	0.294	0.980	0.300	0.150	3.11	0.914	0.750	0.632	1.45	0.032	0.72
0.4	0.7	0.3	0.198	0.389	0.916	0.425	0.212	3.70	1.439	0.912	0.752	1.91	0.057	0.86
0.5	0.8	0.2	0.112	0.475	0.800	0.594	0.297	4.37	2.075	1.097	0.785	2.64	0.109	0.99

*A_c = area corresponding to e_c.

the head of each column. A table of areas and chord lengths of circular segments is all that is required. Supplementing the notation used by the authors, let z = maximum height of the control "hump"; Z = area of the control hump at maximum height; $x = 1 - (y_c + z)$; X = area of a segment with a rise of x; y = depths (y_c, critical depth; y_m, mean depth, etc.); Y = area corresponding to depths, y; t = top width of water surface = chord of a segment with a rise of $z + y_c$; h = velocity head (h_c for critical flow, etc.); ε = energy head above the pipe invert; and A = area above the control corresponding to energy head, ε. All symbols are expressed in terms of the pipe diameter, D.

The depth, y_1 (see Fig. 18), is obtained by deducting the velocity head at this point from the energy head. The trial method used by the authors can be eliminated by dividing the discharge by the area corresponding to the energy head, to obtain the pipe velocity, V_1. This is shown in the last four columns of Table 7. The error involved from assuming no energy loss from y_1 to y_c and that from using the area corresponding to the energy head ε, tend to compensate each other.

The method of computing the rating curve directly illustrated in Table 7 is equally effective for the trapezoidal control described by the authors.

In large sewers in which only the sanitary flow is to be measured, or at manholes over smaller ones, the equipment illustrated in Fig. 19 has been used very effectively. Two screw struts are held in the sewer by expanding them against the sides of the pipe or the manhole. One strut carries a small platform on which the recorder is mounted. The other supports an adjustable pivot from which trails a float that can be adjusted both up stream and down stream and laterally until it is properly adjusted to connect with the style of the recorder.

A scow-shaped float with a rudder was found to have decided advantages over other types. One advantage of this equipment is the ease with which it can be moved from place to place.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

FLOOD AND EROSION CONTROL PROBLEMS AND THEIR SOLUTION

Discussion

BY MESSRS. HARRY F. BLANEY, W. P. ROWE, AND J. B. LIPPINCOTT

HARRY F. BLANEY,[14] M. AM. SOC. C. E. (by letter).[14a]—Some excellent data on débris flows in Southern California are presented by Mr. Eaton. The writer and Colin A. Taylor, Assoc. M. Am. Soc. C. E., made a brief survey[15] of damage resulting from movement of débris out of canyons during the storm of December 30, 1933 to January 1, 1934, described by the author. This incident offers the opportunity to draw several object lessons and to take steps to guard against a recurrence of such losses.

The movement of large masses of detritus out of mountain canyons as mud flows is not an unusual geologic occurrence. Large débris fans have been built up and are steepest and most rugged at the mouths of small canyons in which the stream flow is intermittent. The increasing development of the country has brought more and more of these areas under cultivation, and many of the most desirable residential sections are located high on the fans. However, in many cases, unregulated subdivision has caused the development of entire fans so that the natural watercourses are encroached on before adequate protection is provided.

In mud flows saturated masses of soil furnish the transporting medium that carries huge rocks out of the canyons. The rocks may originate in the same slide with the soil, or they may be picked up in the canyon bottom when temporary dams formed by slides break loose.

A viscous mass of mud slipping from a canyon wall may contain just enough water to make it flow, and progress may be relatively slow in the flatter slopes of the canyon bottom. In this condition, huge rocks are carried along

NOTE.—The paper by E. Courtland Eaton, M. Am. Soc. C. E., was published in September, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1935, by Messrs. Arthur G. Pickett, and R. W. Davenport; and December, 1935, by C. S. Jarvis, M. Am. Soc. C. E.

[14] Irrig. Engr., Bureau of Agricultural Eng., U. S. Dept. of Agriculture, Los Angeles, Calif.

[14a] Received by the Secretary December 19, 1935.

[15] Unpublished report, "Damage Resulting from Movements of Débris Out of Canyons During the Storm of December 30, 1933, to January 1, 1934, in Southern California", by Colin A. Taylor.

without rolling and the movement has the appearance of a lava flow, and, in this condition, may move slowly out of the canyon on to the fan.

When a large quantity of surface run-off occurs at the time the saturated soil slips from the canyon walls, the water may first pass over the top of the slower moving mud flow. Water may also pile up behind the temporary dams and then, as they give way, mix with the soil and make the mass less viscous. As the proportion of water increases the mass becomes more fluid and the rocks tend to settle and roll or slide along the bottom, while the finer material mixed with the water moves over the top of the mass with increasing velocity. The puddled mass that moves the larger boulders drains very slowly.

Eight samples were collected at 4:00 P. M., on January 2, 1934, of the fine material in the wash under some large boulders at Foothill Boulevard, north of Montrose, Calif. (see Fig. 11). The average moisture content was 18.4% by weight, and after screening out the material larger than 2 mm in diameter, the percentage of moisture was 27.8, based on the oven-dry weight of the material smaller than 2 mm. The percentage of water in a saturated soil as it lies in place before it first starts to slip would be approximately 25% by weight, or 33% by volume. This saturated fine material acts as a lubricating medium and assists in the transportation of the larger boulders.

In the canyon bottom mechanical mixing with more water takes place, and the percentage of water must be higher except in the slower moving mud flows. The water content of the material must be continually varying through wide ranges so that the evaluation of flows from gauge heights is most uncertain. There is also the further complication caused by the underlying rocks and viscous layers of material moving much more slowly than the more fluid material on top. Furthermore, the churning bores of flood crests also contain air which tends to increase the maximum gauge height or high-water marks. From this, it follows that the estimation of flows of this character by gauge height record at the ordinary gauging station is most impractical as no record is obtained of the water content of the flow, nor can the velocity be considered as a water equivalent.

It would appear that the most practical method of measuring discharge rates under conditions such as prevailed in these canyons is to gauge the rate of rise in the débris basin at the canyon mouth and also to establish a gauging station in the channel below the débris basin.

One of the most important functions of brush cover in controlling run-off is the aeration of the soil. Decayed roots and worm holes furnish channels through which air is readily moved from the lower soil depths. Rain does not ordinarily run down these channels as is popularly supposed, but moves into the soil as capillary water. The function performed by the roots and worm holes is to release air back to the surface as it is displaced by the downward moving moisture. This release of air prevents the development of back pressure, which would materially reduce the rate of infiltration of rain water, particularly in long-continued steady rains.

The burning of the water-shed increases the potential danger many-fold, but mud flows will occur on certain soil types with the vegetative cover still

intact as witnessed by the flows from Neusbickel Canyon, near San Dimas, Calif., during the same storm. In this case, the soil is a clay loam and the moisture could not be transmitted into the more compact underlying subsoil fast enough to prevent saturation of the top soil. Slips of the top soil then occurred on the steeper slopes. The soil in Neusbickel Canyon supports a dense growth of mustard that starts each winter and dies the following summer. There are also occasional patches of brush. The canyon sides are very steep—60° and more in many places. A 5-in. rain on December 12 to 14, 1933, filled the top soil to field capacity, and the fresh growth of mustard was less than ½ in. high at the start of the storm on December 30, when 15 in. of rain fell within 30 hr. saturating the top soil so that it slipped down the slopes, at many points forming great arrow-shaped scars on the hillside. In some cases, the movement was slow so that the material was carried away as fast as it dropped into the stream bed. In others, the evidence shows that great masses avalanched down and temporarily blocked the canyon. At the end of the storm, however, the stream bed was swept clean to bed-rock and the débris was deposited in a fan-shaped mass at the mouth of the canyon.

The control of moving masses of detritus has been a serious problem in the Western States for many years, and the protection of the irrigated agricultural areas from floods has been the concern of governmental agencies. The problem has been studied in Utah, and control methods have been devised by engineers of the U. S. Bureau of Agricultural Engineering. The results of the Utah investigations were published in 1933.[16] Similar methods of control might be effective in some areas of Southern California for the protection of life and property in the foothill areas.

Mr. Eaton discusses "Spreading to Increase Percolation" and, in Table 13, contributes some valuable information on the effect of scarification by plowing and harrowing on the rates of percolation in the San Gabriel River and Rio Hondo. In this connection, the U. S. Bureau of Agricultural Engineering has been conducting experiments on water-spreading at the mouth of San Gabriel Canyon for several years. This investigation, which is in charge of Mr. A. T. Mitchelson, has produced some interesting data on the stimulating effect of undisturbed native vegetation by root activity on the rate of percolation. In 1930, an experimental plot of 0.38 acre was diked and water was applied uniformly in such a way as not to interfere with the healthy growth of the vegetation. Continuous measurement was made of the water flowing on to the plot and of the run-off. In 1931, a second plot of like size, shape, and soil type was established adjacent to the first, but from it the vegetation was removed, the roots grubbed out, and the surface plowed, harrowed, and furrowed. It was operated simultaneously with the other plot. In 1932, a third plot was established, which was also adjacent to and of the same size as the first, and was operated simultaneously with the others. The third plot, however, was operated on the basin method, a head of about 6 in. of water being held over the wetted area.

[16] "The Barrier System for Control of Floods in Mountain Streams", by L. M. Winsor, Assoc. M. Am. Soc. C. E., *Miscellaneous Publication No. 165*, U. S. Dept. of Agriculture.

In the season, February to June, 1930, during a continuous run of $87\frac{1}{2}$ days, the plot in native vegetation absorbed water at rates averaging 3.98 acre-ft per acre per day and ranging from 3.03 to 5.33 acre-ft per acre per day. In January to March, 1931, during a run of 82 days, this plot absorbed at rates averaging 6.50 acre-ft per acre per day and ranging from 2.13 to 7.92 acre-ft per acre per day, while on the furrowed plot the daily absorption rates averaged only 2.53 acre-ft per acre and ranged from 1.15 to 4.16 acre-ft per acre. In the season, January to June, 1932, the daily percolation rates for the first plot again were highest, averaging 5.03 acre-ft and ranging from 2.12 to 8.04 acre-ft per acre. On the furrowed plot, the rates ranged from 1.66 to 4.22 acre-ft per acre and averaged 2.93 acre-ft per acre per day, while on the basined plot the minimum, maximum, and average rates were, respectively, 2.22, 5.08, and 3.76 acre-ft per acre per day.

W. P. Rowe,[17] Assoc. M. Am. Soc. C. E. (by letter).[17a]—A very complete outline of the flood-control problems of Los Angeles County is presented in this paper. The information contained in Tables 12 and 13, dealing with percolation rates, is of particular interest as millions of dollars are now (1936) being spent in Southern California on projects designed to increase the supply of the underground water basins. In view of these enormous expenditures the writer believes that the economic side of this problem should receive more particular attention.

The paper contains certain statements relative to quantities of material eroded from mountain sides which the writer can not pass unchallenged. This is especially true in view of the widespread interest being displayed in erosion control and the propensities of certain erosion-control enthusiasts in quoting published data to support their claims without analyzing their accuracy.

In discussing this feature of the paper, the writer will use the author's definition, namely: "Erosion, in this instance, is defined as the washing of loose material from weathered mountains and foot-hill areas, and its deposition upon the lower valley floors." Four quantitative estimates of erosion from burned-over areas are cited by Mr. Eaton and he states: "The La Crescenta-Montrose débris flow, on January 1, 1934, furnishes the best and most accurate as well as the largest unit flow of authentic record." Haines Canyon is included in his treatise on this area. Under "Erosion with Normal Water-Shed Cover", he states: "Where water-sheds have not been disturbed by fires, roads, earth-slides, etc., erosion is relatively light", and immediately preceding Fig. 8: "Mass movements originated from canyon sides as high as 200 ft above stream bed, some of the masses being as much as 50 to 150 ft wide. Trees 2 ft, or more, in diameter slipped with the masses and were swept into the streams." It is true that the brush cover on the mountain sides was burned, but most of the trees and other canyon bottom growth escaped unharmed by the fire. This is characteristic of most mountain fires in Southern California. If one were to believe evidences of previous earth

[17] Cons. Engr., San Bernardino, Calif.

[17a] Received by the Secretary December 20, 1935.

slippages in this area and the statement in the paper that this area had not been burned over since 1878, one must conclude that similar slippages would have occurred during the storm of January 1, 1934, regardless of the fire.

It is almost impossible to find a water-shed in Southern California that conforms to the author's description of one on which erosion is relatively light because of the absence of previous "fires, roads, earth-slides, etc." Fig. 13 shows a portion of the San Bernardino Mountains north of

FIG. 13.—EROSION IN THE SAN BERNARDINO MOUNTAINS

San Bernardino. It depicts all the major man-made aids to erosion, namely, earth-slides created by cutting faulted ground, and earth-slides created by the overcast of materials excavated from side-hill cuts, fire-breaks, old roads, and new roads. Most of the area was burned over by fire in 1911 and parts of the same area have been visited by two other fires since that time. The débris removal from the mountain sides by storm waters after the fires was very slight in comparison with that due to the various construction activities. No evil effects from the fires are visible either on the ground, or from the air, and the escape of canyon bottom vegetation from fires is characteristic of Southern California brush fires. The apparent lines of erosion down the canyons is due to the movement of overcast material.

Under "Erosion and Erosion Control", the author states that the forces of erosion are still active and have built deposits of many thousands of

feet of débris in the valleys and he apparently ascribes this deposition to the results of burning of water-shed cover. In his analysis of the quantities of erosion from the San Gabriel Canyon area (see "Erosion with Normal Water-Shed Cover"), he shows that the flood of 1927, one of the highest peak flows of record, caused the erosion of only 3 200 cu yd per sq mile, 3 yr after a major fire. With this as a basis one should assume that it is only a combination of heavy rainfall on a burned-over area that causes débris flow from an undisturbed water-shed. The coincidence of a season of heavy rain following a fire which might occur every 56 yr, as in the case of the La Crescenta-Montrose flood, is too remote to account for the deposition of the enormous valley fill in the La Crescenta-Montrose area, unless one were to reverse the old stand-by theory of the Forest Service propagandists and assume that forest cover no longer makes the rainfall, but that lack of forest cover is responsible for heavy rains. The logical explanation lies in a study of the geology of the water-shed by which it will be found that erosion has been active regardless of fires.

Following Fig. 8 the author, describing the movement of peak flows of débris and water below the canyon mouths, states: "Reaching open territory these waves flattened to 5 or 6 ft in height and spread to widths of 100 to 200 ft, scouring new channels and thus picking up new débris loads"; then (preceding Table 10): "Each sharp peak of major débris flow would be succeeded by a rapidly moving stream which contained less débris, cutting a channel to one side or through the deposited débris masses"; and, again (following Fig. 11): "* * * but the final lowest channel section as left resulted from cutting by the later and receding stream flows of higher velocities, which ground the stream bed far below its original grade and below that at the time of the making of the highest mud marks."

The débris cones on which this channel cutting and shifting occurred are the result of previous erosions and the re-working of this material can not be classed as erosion under the author's definition. Nevertheless, he includes the quantity of this re-eroded material in his estimates of erosion per square mile of water-shed.

The author has failed to mention one very important factor in describing the shifting channels and later cutting on the La Crescenta-Montrose débris cones. Prior to the flood, a series of wire and rock check dams had been constructed across the active stream channels. The crests of these dams in many instances were less than a foot below the top of the banks on either side. When the flood flows from the mountain occurred, they overtopped these dams and scoured out the loose alluvial material below them and deposited it behind the dam next lower down stream. When the small basins behind these dams were filled to the crests of the dams, the water and débris were diverted laterally around the ends of the dams and on to the adjacent territory. The $59\frac{1}{2}$-ton boulder mentioned by the author (see following Fig. 9) was deposited in this manner. It was not carried from the mountain side by this one flood, but rolled down the débris-filled channel until it was diverted by one of these check dams. The remnants of the check dams in

this vicinity gave ample proof of this theory. The fact that the active channels on these débris cones usually ride on the highest part of the cones aided in this channel shifting.

The writer will agree with the author in the statement as to the effectiveness of the Haines Canyon débris basin in controlling débris flows, but disagrees when he attributes the 32 000 cu yd of débris caught in the basin to erosion from the mountain side under his definition. This basin was the result of the operation of a gravel company excavating at the head of the débris cone of Haines Canyon at its apex within the canyon walls. The original active channel had a slope of 10 per cent. The operation of the gravel company in excavating below the original grade of the stream resulted in a face at least 40 ft high at the upper end. This face was practically vertical and consisted of sand, gravel, and boulders. When the flood occurred, the water poured over this face and started to re-adjust the grade of its bed to fit this new condition. The result was a cutting of a deep channel up stream from the gravel pit, all the eroded material being deposited in the basin. The classification of all the material deposited in the débris basin as erosion from 0.47 sq mile of burned-over water-shed and the assumption that, therefore, the rate of erosion from 1 sq mile of burned-over water-shed was 62 000 cu yd, would not conform to the author's own definition.

Fig. 14 is a view looking up stream across the Haines Canyon débris basin on March 19, 1934. It shows (1) the re-adjusted stream bed with banks from

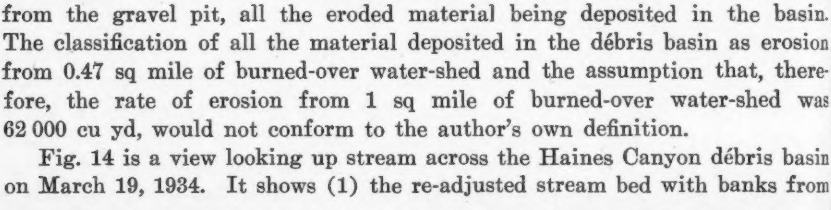


Fig. 14.—Débris Basin in Haines Canyon on March 19, 1934.

15 to 30 ft high; (2) the débris cone deposited within the basin, but above the spillway level; (3) the original high-water line at the top edge of the recent excavation made after the storm of January 1; (4) a re-adjustment of grade on the old cone by stream flow by a storm after January 1; and (5) the extent of the most recent excavation. The spillway is to the right of

the photograph, and its crest is about 10 ft below the level of the remnant of narrow-gauge railroad showing at the foot of the bank.

The importance of having the spillway of a débris basin in solid cut has been shown by the author; but of equal importance is the protection of the inlet channel so that the basin will not be filled from stream-bed deposits washing in as the stream re-adjusts its grade to fit the new conditions.

The evils of erosion are bad enough without undue magnification. The writer is probably old-fashioned in his beliefs, but he still maintains that the Grand Canyon was not the result of forest fires, draining of swamps, or any of the other multitudes of natural evils that have been blamed on the activities of mankind.

J. B. Lippincott,[18] M. Am. Soc. C. E. (by letter).[18a]—Many interesting data have been collected in this paper with reference to floods and their resulting erosion. Of these two problems the greater is erosion. This information is of value because, since the formation of the Los Angeles County Flood Control District in 1915, there have been few if any engineering reports published containing the physical data collected, or of construction progress, other than through the daily press, despite the request of the local engineers for such information. By some peculiar twist of the legal mind the District Attorneys ruled that bond funds could not be so expended. This paper, therefore, is welcome to engineers. A total of $46 308 044, from various sources, had been spent on this work prior to the fall of 1935. Since then $10 000 000 more have been allotted to the District from Federal sources.

Since the appointment of C. H. Howell, Assoc. M. Am. Soc. C. E., as Chief Engineer on January 30, 1935, a comprehensive document entitled "Rainfall and Runoff Report, Seasons 1932–33 and 1933–34" was issued under date of June 1, 1935, which contains hydrographic data for that period. Especially in view of the difficulties of the problems involved it is hoped that, in the future, reports will be currently presented relative not only to hydrography, but also to the engineering features, including the reports of boards of engineers and geologists. This is in accordance with ordinary practice for large public works.

Bonds were voted by the Los Angeles County Flood Control District for two purposes: (1) The prevention of flood damage; and (2) the conservation of water by spreading floods over absorbent valley fills for underground storage. The general plan contemplates regulating reservoirs in the mountains, and flood-control channels across the valleys and plains, supplemented by spreading basins by means of which the water could be saved. The building of the dams has been difficult because of the geological character of the rocks of the Sierra Madre Range. These rocks have been shattered by earth movements and largely decomposed. The foundations and abutments for these dams have been poor. The local engineers are not alone in having difficulties with these adverse conditions. Engineers and geologists from numerous other parts of the United States have been consulted, with much the same experi-

[18] Cons. Hydr. Engr., Los Angeles, Calif.

[18a] Received by the Secretary January 11, 1936.

ence. Twelve dams have been built successfully but there have been several disappointments. Important features of the general problem are yet unsolved. The dense population of Los Angeles County and the high property values (both of which are threatened) create an insistent demand for protection. Very little has been done in building spreading basins although the Supreme Court of California has pronounced this as one of the main features of the project.

Because of the steepness of the mountain drainage basins, violent, flashy floods are discharged on to the valleys at intervals averaging about once in five years. These floods are followed by long periods of low flow. As described by the author, they carry surprisingly large quantities of débris. During an occasion of this nature the rolling and grinding of the boulders can sometimes be heard for several miles.

The gradient of the San Gabriel River, which is the principal stream of Los Angeles County, indicates the difficulty of the problem of disposing of this débris-laden water. Its smaller tributary basins have stream-bed slopes of from 125 to 400 ft per mile, and the main channel of the river in its mountain section, has a slope of 60 ft per mile. Below the mouths of the canyon the gradient across the San Gabriel Valley for a distance of 12 miles is 25 ft per mile. The river then flows through a pass on to the Coastal Plain where for a distance to the sea of 20 miles the gradient is 6.5 ft per mile. The hydraulic properties of these channels, even when leveed, become more unfavorable at points nearer the ocean. In the case of the Verdugo Wash (north of the City of Los Angeles), the gradients across its débris cones at La Crescenta (Montrose) are 500 ft per mile; thence, through the City of Glendale, the channel has a grade of 80 ft per mile, discharging into the Los Angeles River in the City of Los Angeles, where the channel has a slope of 25 ft per mile. The discharge of mountain floods ranging from 500 to 1 000 sec-ft per sq mile, eroding the shattered and decomposed rocks, has resulted in the building up of débris cones of great volume at the mouths of the canyons, and a general delta plain toward the sea. These débris and delta formations are of universal occurrence. Topographically, many of the cones are surprisingly symmetrical. All the channels are unstable and shifting. The steep slope and the great volumes of the débris cones indicate that the process of their formation has extended through long periods of time and that these deposits will continue.

Because of the commanding elevations of these cones and their attractive climate they have been extensively occupied and improved and, in many instances, towns are located in these exposed locations. The author shows that where the small mountain basins have been recently denuded by fire (which cannot be entirely avoided during the long dry summers) the volumes eroded from the smaller and steeper basins amount to extremes of from 40 000 to 50 000 cu yd per sq mile during single floods. If the floods are diverted at the mouths of the canyons, into lined channels, as is being done, with the foregoing gradients, the place where the débris will be dropped depends on the flattening of the gradient. The deposit must occur somewhere before the flood reaches the sea. As an indication of this last-named process apparently

the mouth of the Los Angeles River has changed its position 18 miles, the San Gabriel River, 9 miles, and the Santa Ana River, in Orange County, 10 miles.

Efforts to Solve the Problem.—As stated previously, the population and property value of Los Angeles County is such that efforts must be continued to control both the flood and the débris. The first general plan of attack was made in 1917 when $4 450 000 in bonds was voted for the building of dams on the Arroyo Seco and San Dimas River, together with the diversion of the flood waters of the Los Angeles River from Los Angeles Harbor.

This was followed in 1924 by a plan that was hurriedly presented, without mature engineering study, and against the protest of the Los Angeles Section of the Society. It involved an expenditure of $35 300 000 for building dams, channel improvements, and spreading basins, the announced program being to regulate the floods of the major streams and to put the regulated flows into underground storage.

Despite the recommendation of two boards appointed subsequent to the bond election (relative to the original San Gabriel Dam), the Courts held that the type, size, and location of the structure defined in the bond election proceedings had to be followed rigidly. After foundation difficulties had developed in connection with the construction of this major structure, the California Supreme Court subsequently held that, because of "changed conditions", modification in the plans could be adopted, an essential feature in the decision being that the resulting decreased surface storage should be replaced by an increase in the quantity of water to be stored underground.

Although to date (1936) twelve dams have been built and are performing service of value, other efforts to build large dams have been unsatisfactory in some instances.

As the smaller reservoirs are practically débris basins in which to impound eroded material, the cost of some of them as such is of interest. The costs in Table 15 are high but they are less than the expense of $1.00

TABLE 15.—Costs of Small Reservoirs

Name of dam	Reservoir capacity, in acre-feet	Storage Cost	
		Per cubic yard	Per acre-foot
Live Oak	250	$0.47	$760
Sawpit	470	0.83	1 340
Little Santa Anita	110	0.79	1 280
Big Dalton	1 290	0.49	790

per cu yd for cleaning up débris that was deposited at the rate of 26 000 cu yd per sq mile of drainage basin in October, 1928, on to streets and yards below the mouth of Brand Canyon, in Glendale, Calif., as stated by the author. His description of these débris flows is especially interesting and presents data that are new. It should be remembered that the great débris flows occur only when the burning of the cover of the drainage basin is followed by torrential rains. Although this combination may occur only at long intervals (say, 10 to 15 yr), the menace is present constantly.

Check Dams.—During the early years of the District there was a popular demand for the building of "check dams." This idea extended throughout Southern California. These check dams at first consisted of dry rock-fills, about 5 to 10 ft in height, built in the mountain channels without being founded on bed-rock. Frequently, they were only a few hundred feet apart, the idea being that their small basins would fill with débris which would absorb the flood water and permit of slow percolation later in the season, and also that they would step down the steep gradients of the channel. These structures promptly failed with the first floods. They were followed by a modified construction, wrapping loose rock in wire mesh; 4 060 of these check dams were built by the Flood Control District. They were also placed a few hundred feet apart in the canyons and on débris cones. The rolling of large rocks by floods cut the wires, and the floods also washed around their ends. To date, approximately 2 000 of these dams have failed under flood action. Accumulated material behind them contributed to débris that was projected on to underlying areas. It was stated that "check dams" had been used successfully in the Alps. Observations in Switzerland and Japan where flood and débris problems prevail, are that such small dams are substantial structures built of cut stone or cement masonry, and are usually founded upon bed-rock. The Los Angeles County check dams were built largely against the protests of the engineers in charge of flood-control work.[19]

Although there are 1 589 sq miles of mountain drainage area in the Flood Control District, the writer knows of no comprehensive estimate of the cost of controlling floods in this area by check dams. Some cost figures, therefore, may be interesting. The flood-control engineers have reported[20] that "the cost per cubic yard of debris stored by check dams varies widely, ranging from as low as 45 cents per cubic yard of debris up to as high as $6.00." If a cost of $3.00 per cu yd for such storage is assumed, this is equivalent to $4 839 per acre-ft of capacity. The following is also given from a report[20] written in 1931: "The actual measurements from a single moderate storm where fires have denuded the watershed have shown over 25 000 cubic yards of débris from one square mile of foothill area with a possible recurrence equal in amount for each storm during the period required for regrowth of vegetation." This re-growth requires fifteen years. Horse Canyon, a tributary of the San Gabriel River having 2.15 sq miles of drainage was completely "check dammed" in the summer of 1932. Twenty-three rock and wire check dams were built, and 137 loose rock dams, which is a total of 160 check dams, or 1 check dam for each 6.25 acres. The cost per square mile of this work was $10 445, and the cost per cubic yard of débris storage, 51 cents. An estimate of the cost of controlling 13.76 sq miles in this basin was $256 082, or an average of $18 610 per sq mile.[21] The conclusion is reached that the concrete dams and reservoirs, such as those referred to previously, none of which has failed, are more substantial and no more

[19] Rept. of E. Courtland Eaton, M. Am. Soc. C. E., August 17, 1927.

[20] Rept. by E. Courtland Eaton, M. Am. Soc. C. E., and Frank Gillelen, Assoc. M. Am. Soc. C. E., May 22, 1931.

[21] Rept. by E. Courtland Eaton, M. Am. Soc. C. E., October 31, 1932.

expensive than the "check dams." Studies have been made in Santa Barbara County, California, by the U. S. Forest Service for controlling the débris on the Santa Ynez, which confirm these conclusions.

Flood-control work in Switzerland by means of larger and more substantial check dams on Lambac Creek, near Brienze, having a drainage area of 1.66 sq miles, shows an expenditure between 1898 and 1913 of $216 000, of which 60% was for channel construction below the mouth of the canyon. This amounts to a cost of $130 000 per sq mile. At the time of this cost estimate the work had not been completed. Where suitable reservoir and dam sites exist on larger drainage areas, the construction of the larger and more substantial dams is justified.

Débris Basins.—The next effort by the Flood Control District in the development of a plan to control débris from small mountain drainage areas was by means of "débris basins." These basins are excavated in the débris cone, with overflow weirs discharging into lined channels. The basins are built at the mouths of canyons at the apex of the débris cone. The general theory is to obtain a storage capacity of 100 000 cu yd of débris for each square mile of tributary drainage basin. The excavated material consists of sand, gravel, and large boulders. The plan is to discharge the floods through these basins in which the larger débris will be deposited and build concrete-lined channels of large capacity from the outlet of the basin down the slopes to the larger drainage lines and thence through other larger lined channels to the rivers. To January 20, 1936, twenty-three of these basins have been built.

The following is a description by the Flood Control engineers of one of these basins in the Montrose District, called Haines Canyon: The drainage area, in square miles, is 1.50. Of the basin, 60% has been burned. The present water capacity of the basin as built is 24 700 cu yd. Assuming that the débris in the basin itself takes a slope of approximately one-half the natural slope of the channel, its débris capacity is estimated at 87 250 cu yd. It is proposed to enlarge this débris capacity ultimately to 150 000 cu yd. What is termed the "bulked Q", of water and débris, in cubic feet per second, is 200% of the maximum peak water flow calculated for the drainage area above the basin. This bulked Q is given as 10 000 cu ft per sec. This will allow bulking by débris of 100%, whereas the estimates of débris flow for the January 1, 1934, storm in the La Crescenta area, indicated a bulking of 50% by débris. What is called the "clear-water Q" of 5 000 sec-ft is the design Q, for the first section of the channel below the basin. In designing the inlet and outlet structure of the basin, the bulked Q is used.

As the author well shows the débris brought down by floods is enormously increased when such floods follow soon after the burning of the brush cover of the basins. He states that following a fire, the floods of January 1, 1934, discharged from 7 sq miles of drainage north of La Crescenta (Montrose) 659 000 cu yd on to property and streets, amounting to more than 90 000 cu yd per sq mile. Other instances are given by the author of greater débris flow from other burnt areas. Referring to the cost of these débris basins he states: "Their capital costs per cubic yard of débris capacity will range from

$0.30 to $1.00 per cu yd, depending mainly upon excavation costs." It will be noted from these figures that the cost of débris-basin storage does not differ greatly from the costs of similar storage behind the four concrete dams previously cited.

The probable life of the débris basin as well as of the small reservoir will be short. In the case of the débris basins the plan is to re-excavate them when they are partly or entirely filled. When the smaller reservoir is filled the probable plan may be to provide another on the same stream. These figures are presented not in criticism of the plans of the Flood Control District, but rather as indicative of the expensive nature of the problem involved and which should be solved. These great débris flows occur only when floods follow fires, a combination of events that does not occur frequently. Nevertheless, they are a menace that should be provided against.

Assuming maximum flood discharges of 500 to 1 000 cu ft per sq mile delivered into concrete-lined channels having gradients of more than 10%, the further question arises as to whether the exceedingly high-water velocities of fully 40 ft per sec which may occur, will flow around curves in the channels irrespective of their débris. The opportunity to observe this phenomenon has not yet occurred.

Brush and Forest Cover.—The beneficial effect of forest or brush cover in reducing floods and erosion has been amply demonstrated in California by observation on adjacent experimental plots both burnt and unburnt in two mountainous regions, and also by careful observations on neighboring drainage basins of similar exposure some of which have been denuded by fire. These measurements (which are in accord) have been conducted by the U. S. Department of Agriculture. To give the details would be to repeat the views of the writer which have been presented[22] previously. They show in one series of test plots that the run-off from burnt areas, with rain at the rate of 2.4 in. per hr, was 200 cu ft per sec per sq mile and that there was "no surficial run-off measured" from the covered plots. Eroded material from the burnt plots was 3 cu yd per acre and that from the unburnt area was zero. With a 12-in. rainfall during the storm of December 30, 1933, to January 1, 1934, the peak run-off from the Verdugo Wash of 19.3 sq miles (which included the Montrose area, 6 sq miles of which was steep, absorbent, canyon floor and largely burnt over), was 350 cu ft per sec per sq mile and the erosion, 50 000 cu yd per sq mile, whereas, from the unburnt mountainous adjacent Arroyo Seco Basin of 16.24 sq miles with brush cover, the run-off was 58 cu ft per sec per sq mile and the erosion was relatively insignificant. Other basins show greater contrasts.

These examples show the great value of the dense brush on the Southern California mountains. If this cover can be maintained, it relieves the flood and débris problem. If fires are prevented it affords a much simpler and less expensive process than the building of costly engineering works. This solution resolves itself into the effort to prevent fires. The Federal, State, and County organizations have combined to prevent fires and have succeeded substantially in reducing their hazard. Lookouts with telephone connections have been

[22] *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 330.

established throughout the mountains. Fire-breaks have been built along ridges. The system of roads being built, will permit of the rapid transportation of men and equipment to fires. If a fire can be reached in its incipiency, it can be extinguished. If, however, it becomes general, the task is well-nigh hopeless; the effort then is to control it within drainage basins. In the writer's opinion relatively greater benefits can be obtained from efforts to prevent fires than by the construction of engineering works, although both are necessary. A most rigid patrol and control of the use of the Forest Reserve is justified.

The occupation of the menaced débris cones should be discouraged and should have been prohibited. However, they are already (1936) largely occupied. As stated by the author this occupation invites disasters such as that which occurred on January 1, 1934, when 30 people lost their lives and $5 000 000 property damage occurred at Montrose. It is unfair to require the public to pay the costs for their protection which may be at a money cost in excess of assessed values. The location of these improvements in many instances prevents the spreading of flood water for conservation purposes on the cones as originally planned.

Conclusions.—In conclusion, the writer expresses his appreciation of the difficulties involved in the handling of the flood problems of the Los Angeles County Flood Control District. The task appears well-nigh impossible of complete solution on the one hand and a necessity on the other. The presentation of basic data in this paper is valuable. Further reports on the engineering problems of the District should be made generally available in order that the minds of many engineers can give it thought. The engineers in charge of the work are entitled to helpful co-operation. Opportunities for conserving the valuable flood water by its underground storage are yet available in the larger valleys and on the Coastal Plain and should not be neglected as this is one of the main features of the project. It is a cheaper conservation method in Los Angeles County than the storing of water in surface reservoirs. However, each drainage basin requires a separate study and plan.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

TUNNEL AND PENSTOCK TESTS AT CHELAN STATION, WASHINGTON

Discussion

BY W. A. HILL, ESQ.

W. A. HILL,[3] ESQ. (by letter).[3a]—An important addition to the data available concerning the hydraulic behavior of large pressure conduits in actual operation, is contained in this paper. Although it is to be regretted that no piezometer connection existed immediately above the wye branch, for the purpose of checking the wye-branch losses and, at the same time, affording a direct means of determining the friction losses in the steel-lined section of the tunnel, the conformity of the test points, to the theory of the nature and location of the wye-branch losses, is such that the validity of the analysis can scarcely be questioned.

The values of the coefficient, C_w, as derived, furnish an important confirmation of those already in use for the concrete section of the pressure tunnel; but the apparent exponential discrepancy, with respect to the steel-lined section, is unexplained. The additional point of observation, previously mentioned, would have aided materially in substantiating these results.

The most illuminating part of the analysis is that which deals with the losses occurring in the vicinity of the wye branch. Little work has been done with this special type of structure in the past, with a view to determining its behavior under varying conditions of flow, and the magnitude of the losses involved. Mr. Fosdick segregates these losses into two classes, "eddy" losses and "diversion" losses; the former take place above the point of bifurcation and affect both penstock branches alike, whereas the latter occur beyond this point and reflect the energy losses in the individual branches only. Due to the destruction of directional velocity head, both are essentially impact losses, but their respective values are widely disproportional to the velocity heads involved. A comparison of velocity-head changes produced (see Table 2)—with corre-

NOTE.—The paper by Ellery R. Fosdick, Esq., was published in October, 1935, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[3] Hydr. Engr., The Washington Water Power Co., Spokane, Wash.

[3a] Received by the Secretary January 3, 1936.

sponding losses observed, throughout the range of one unit shut down and the other increasing load—will illustrate this point clearly. For the range selected, $\Delta h_v = h_v$ (since $h_{v2} = 0$), and $\Delta h_{v1} = h_{v1} \sin^2 \alpha$ ($\alpha = 22° \ 30'$).

TABLE 2.—Comparison of Velocity-Head Changes and Corresponding Losses.

Q	Δh_v	h_e	Factor	Δh_{v1}	h_α	Factor
200	0.04	0.01	0.03	0.02	0.10	5.0
400	0.16	0.02	0.10	0.10	0.50	5.0
600	0.37	0.08	0.22	0.21	1.20	5.7
800	0.66	0.26	0.39	0.38	2.10	5.5
1 000	1.03	0.59	0.57	0.60	3.10	5.2
1 160	1.38	1.03	0.74	0.80	3.90	4.9

A glance at the respective factors for eddy and diversion losses in Table 2 reveals that the former never equal unity, indicating a partial recovery of velocity head; in the case of the diversion losses, however, the approximate factor of five indicates a loss far in excess of the total theoretical impact values.

The obvious disparity in the relationship of these losses may lie, conceivably, wholly in the fact that all values of velocity heads used in the computations are based on average conduit velocities, whereas the observed effects result from the actual velocities at the points of varying influence. The velocity changes which produce the "eddy" losses may occur largely near the wall of the conduit, the higher interior velocities being gradually diverted toward the branch carrying the larger flow. That an opposite condition exists with respect to the "diversion" losses is a certainty, for inspection of the design shows that the greatest and most abrupt diversion of flow occurs on the produced axis of the pressure tunnel, where the velocity is at its maximum value. The conclusion to be drawn from these facts is that attempted computation of losses in structures of this character should be made only after giving careful consideration to the matter of velocity distribution.

The magnitude of the losses evident in this particular installation arrests the attention indeed. When a total head loss of 5 ft for one unit wide open, or of almost 4 ft for full plant load, occurs in such a short length of flow line, it is certainly time to give some attention to the matter of design. A quick computation of the horse-power equivalent of this loss will give some idea of its economic importance. However, as long as the problem is left solely in the hands of the structural designer, little, if any, improvement may be expected.

The "eddy" losses, characteristic of unbalanced flow, can never be eliminated entirely although their maximum effect can probably be considerably reduced; the "diversion" losses, which are by far the more serious, can be minimized by adequate care in design. The structural problem involved can be solved in several ways, the method to be used depending on such physical characteristics as internal pressure, size of conduit, and location of structure.

One method of combining hydraulic efficiency with structural adequacy, which is seldom, if ever, used in the United States, is that developed by the

French engineer, M. Ferrand; for simplicity and effectiveness it is unexcelled. The device consists, essentially, of a manifold, of proper hydraulic design, which is entirely inclosed in a steel jacket of appropriate structural capacity. This jacket may be of spherical form, of combined cylindrical and hemi-spherical form, or of combined conical and hemi-spherical form, depending upon the dimensions of the inner structure to be housed. By means of open ports in the main penstock, the conduit pressure at any instant is immediately communicated to the otherwise closed chamber lying between the outer shell and the manifold proper. In this manner a uniform, and always balanced, support is furnished for the interior or water-conducting shape. It will be evident that the duplication of metal involved is of little moment, for the interior structure need be only of sufficient weight to withstand unbalanced kinetic pressures.

In view of the fact that the science of hydraulics is, and ever must be, in a large measure dependent upon empiricism, it is to be hoped that a keener appreciation of the certain benefit to be derived from the equipping and testing of future installations, will lead those responsible for the design and construction of hydraulic projects to incorporate in their plans provision for further work of this character. As a general rule, the expense involved in adapting a hydraulic structure to the rôle of laboratory model is so trifling, as compared with the value of the data that may be secured in this manner, that failure to do so can be construed only as a mark of professional indifference.

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
DISCUSSIONS

TAPERED STRUCTURAL MEMBERS: AN ANALYTICAL TREATMENT

Discussion

BY MESSRS. FRED L. PLUMMER, AND LEROY W. CLARK

FRED L. PLUMMER,[5] ASSOC. M. AM. SOC. C. E. (by letter).[5a]—In an effective manner the authors illustrate a valuable, although not new, method of expressing mathematically the approximate variation in moment of inertia of structural members of variable cross-section. The form of the empirical equation is such that the integrations required for the solution of most problems involving deflections or redundant forces in structures consisting of members subjected to bending moments, can be performed very easily. In 1925, the writer developed a modified form of the slope deflection method which could be used for members of variable cross-section (printed in blue-print form only). Formulas corresponding to Equation (31) to Equation (47) were developed. In applying these formulas to specific types of members the true variation in moment of inertia was used in many cases, even though the resulting integrations were quite difficult. In a few cases a variation identical with that proposed by the authors was used.

Unfortunately, many engineers in design offices are unwilling or unable to carry through the very simple integrations which are required when the method suggested by the authors is followed. Formulas or tabulated coefficients must be made available before such engineers, many of whom have had little or no training in the analysis of continuous structures, can be expected to use such structures and to design them easily and competently. These men will welcome the formulas developed by the authors. Extensive tables of coefficients have been prepared and published by L. T. Evans[6], Jun. Am. Soc. C. E., E. B. Russell[7], T. F. Hickerson[8], M. Am. Soc. C. E., and others. The mere availability of tables and formulas giving coefficients for tapered members, however, will not produce well designed structures. Such aids are only tools to help eliminate some of the analysis routine. The values depend

NOTE.—The paper by Walter H. Weiskopf and John W. Pickworth, Assoc. Members, Am. Soc. C. E., was published in October, 1935, *Proceedings*. This discussion is published in *Proceedings* in order that the views expressed may be brought before all members for further discusion of the paper.

[5] Assoc. Prof., Structural Eng., Case School of Applied Science, Cleveland, Ohio.

[5a] Received by the Secretary December 20, 1935.

[6] "Handbook of Rigid Frame Analysis", 1934, Edwards Bros., Inc.

[7] "Analysis of Continuous Frames", by Ellison and Russell, 1934.

[8] "Structural Frameworks", 1934, Univ. of North Carolina Press.

upon definite assumptions[9] which must be recognized and given consideration when the designing engineer attempts to predict how a given structure will act and what stresses will probably exist when such action takes place.

A number of tables containing constants for use in designing tapered beams[10] has been produced by A. Strassner. These values were revised by Walter Ruppel, M. Am. Soc. C. E., and printed in a form suitable for use in connection with the conjugate and fixed-point methods of analysis.[11] The same values are equally important in connection with other methods since they represent some of the general integrals set up by the authors. Six sets of values are given by the tables for various types of members and kinds of loads. The tables cover several forms of members and types of loading. Equations (52), (53), (61), (62), (110), and (111) of the paper may be extended, as follows, respectively:

$$C_{AB} = -\frac{F_1}{F_2} = \frac{-v}{1-v} \quad \ldots\ldots\ldots\ldots\ldots\ldots (140)$$

$$C_{BA} = -\frac{F_1}{F_3} = \frac{-u}{1-u} \quad \ldots\ldots\ldots\ldots\ldots\ldots (141)$$

$$S_A = \frac{I_A}{4\,L}\,\frac{F_2}{(F_2\,F_3 - F^2_1)} = \frac{I}{4\,L}\,\frac{1-v}{p\,(1-u-v)} \ldots\ldots\ldots (142)$$

$$S_B = \frac{I_A}{4\,L}\,\frac{F_3}{(F_2\,F_3 - F^2_1)} = \frac{I}{4\,L}\,\frac{1-u}{q\,(1-u-v)} \quad \ldots\ldots (143)$$

$$S'_A = \frac{I_A}{4\,L\,F_3} = \frac{I}{4\,L\,p\,(1-u)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (144)$$

and,

$$S'_B = \frac{I_A}{4\,L\,F_2} = \frac{I}{4\,L\,q\,(1-v)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (145)$$

In Equations (140) to (145), I is the minimum value for the members and is not necessarily equal to I_A, the moment of inertia at the left end of the member. Furthermore (differing from the notation of the paper),

$$u = \frac{\dfrac{1}{L}\displaystyle\int_0^L \frac{(L-x)\,x\,dx}{Z}}{\displaystyle\int_0^L \frac{(L-x)\,dx}{Z}}$$

and,

$$v = \frac{\dfrac{1}{L}\displaystyle\int_0^L \frac{(L-x)\,x\,dx}{Z}}{\displaystyle\int_0^L \frac{x\,dx}{Z}}$$

[9] "Continuous Frames of Reinforced Concrete", 1932, John Wiley & Sons, Inc.

[10] "Neuere Methoden", Vol. 1, Second Edition, Berlin, 1921.

[11] *Transactions*, Am. Soc. C. E., Vol. 90 (1927), p. 167, *et seq.*

The writer has reproduced and demonstrated the use of these same constants in connection with the slope deflection, moment distribution, and other methods.[12] While these tables cover a wide range of conditions, the original tables by Strassner are even more extensive and include all forms that are commonly used in engineering structures.

Any engineer who attempts to develop additional tables, however, will find that the method suggested by the authors will very materially facilitate his work and, at the same time, result in no appreciable loss of accuracy.

LeRoy W. Clark,[13] M. Am. Soc. C. E. (by letter).[13a]—The method of analyzing tapered members presented by the authors is interesting and ingenious, but the writer doubts whether it will be generally adopted. To test its usefulness he assigned to a group of graduate students some problems using haunched beams, and found that except in a few cases they could obtain correct results in less time by using the column analogy[14], while the substitute *I*-curve gave results which were very unreliable.

For beams with moderate haunches the results agree very well with correct values. For the beam, as shown in Fig. 13(*a*), using the authors' suggested

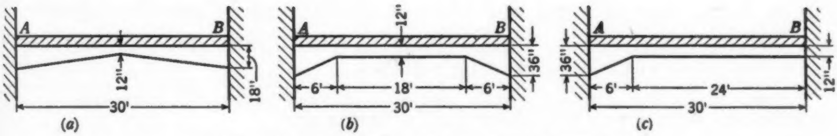


FIG. 13.—EXAMPLES OF TAPERED MEMBERS.

approximate value of 2 for the shape modulus, *n* gives results not exceeding 8% in error. With *n* based on *I* at a distance of 7.5 ft from End *A*, the errors do not exceed $2\frac{1}{2}$ per cent.

For beams with deep short haunches the method is not so satisfactory. Computations were made for the beam shown in Fig. 13(*b*). The shape exponent was taken first as 2, and then calculated for the moments of inertia at distances of 1, 2, 3, 4, and 5 ft from End *A*. The percentage of error in each

TABLE 1.—PERCENTAGE OF ERROR, WEISKOPF-PICKWORTH ANALYSIS

Values of *n*	Bending moment, *M*.	Stiffness factor, *S*.	Carry-over factor, *C*.
2	4.8	146	9.3
Values of *n* corresponding to the moment of inertia at:			
$x = 1$ ft.	2.4	85	4.9
$x = 2$ ft.	2.6	90	5.3
$x = 3$ ft.	2.2	81	4.5
$x = 4$ ft.	0.7	55	1.9
$x = 5$ ft.	—3.5	7	—5.3

case is shown in Table 1. If *n* is computed for the moment of inertia at a distance of 6 ft, the results are the same as those for a prismatic beam of uniform depth of 1 ft.

[12] "Statically Indeterminate Structures", 1934, Edwards Bros., Inc.

[13] Prof. of Mechanics, Rensselaer Polytechnic Inst.; Cons. Engr., Troy, N. Y.

[13a] Received by the Secretary December 28, 1935.

[14] *Bulletin 215*, Eng. Experiment Station, Univ. of Illinois, Urbana, Ill.

In the case of an unsymmetrical beam of the dimensions shown in Fig. 13(c), an attempt to solve the problem by using one section for the entire beam and a value of $n = 2$, gave absurd results, as might have been expected With this solution, using a value of n based on the moment of inertia at $x = 3$ ft, the stiffness factors at Ends A and B were 110% and 25% in error. respectively. Using two I-curves reduced the maximum error to about 4%, but the labor involved is at least as great as in the column analogy. It is evident that the greatest errors always occur in the stiffness factor, S.

In practically all the problems solved the results were considerably in error, and the errors varied widely, depending upon the point through which the substitute I-curve is made to pass. This is a matter of judgment, and a designer with limited experience would have no criterion by which to judge the correctness of his work, whereas he could have absolute confidence in the results of the column analogy. Although an experienced designer could doubtless use the substitute I-curve satisfactorily, it is doubtful whether he would save much time in so doing, and in all probability he would have constructed curves from which the desired results could be read easily. Such a series of curves was prepared by Mr. R. H. Trathen, in "A Study of the Effect of Beam Haunching on Fixed End Moment, Stiffness, and Carry-Over Factor."[15]

The authors should be complimented for an extremely well written, and a clear presentation of an interesting solution.

[15] Thesis presented to Rensselaer Polytechnic Inst. in partial fulfillment of the requirements for the degree of Master of Civil Engineering.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

PROPOSED IMPROVEMENT OF THE CAPE COD CANAL

Discussion

BY GEORGE R. RICH, M. AM. SOC. C. E.

GEORGE R. RICH,[14] M. AM. SOC. C. E. (by letter).[14a]—The application of the harmonic theory advanced by the late General Parsons[2], or the reflected wave theory defined by Colonel Brown[5] to the determination of velocities and surface elevations at several selected cross-sections of an open waterway, such as the Cape Cod Canal, connecting two seas having substantial differences in tidal phase and amplitude, is both complicated and laborious. Moreover, as stated by Captain Harwood, such involved calculation is unwarranted in many cases because of the paucity of available data upon which to base the selection of the pertinent physical constants.

However, in computing the action of a navigation lock, located some appreciable distance from the entrance to Cape Cod Bay, in raising the elevation of high tide and depressing the elevation of low tide at the lock site, relative to similar Cape Cod Bay levels, the writer has found that the basic equations of General Parsons may be adapted to give a rational yet comparatively simple means of estimating the hydraulic effect on the canal in advance of conducting confirmatory model tests. For example, the two general equations defining the oscillatory flow are:

$$\xi = e^{p x}\left[C \cos\left(\frac{2 \pi t}{T} + q x\right) + D \sin\left(\frac{2 \pi t}{T} + q x\right)\right] + e^{-p x}\left[C' \cos\left(\frac{2 \pi t}{T} - q x\right) - D' \sin\left(\frac{2 \pi t}{T} - q x\right)\right] \quad \ldots\ldots(6)$$

NOTE.—The paper by Capt. E. C. Harwood, Corps. of Engrs., U. S. A., was published in October, 1935, *Proceedings*. Discussion of this paper has appeared in *Proceedings*, as follows: December, 1935, by C. S. Jarvis, M. Am. Soc. C. E.

[14] Hydr. Engr., Passamaquoddy Tidal Power Project, U. S. Engr. Office, Eastport, Me.

[14a] Received by the Secretary November 16, 1935.

[2] "The Cape Cod Canal", by William Barclay Parsons, Hon. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 1.

[5] "Flow of Water in Tidal Canals", by Earl I. Brown, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 749.

and,

$$h = -d\,e^{px}\left[(C\,p + D\,q)\cos\left(\frac{2\,\pi\,t}{T} + q\,x\right) + (D\,p - C\,q)\sin\left(\frac{2\,\pi\,t}{T} + q\,x\right)\right] + d\,e^{-px}\left[(C'\,p - D'\,q)\cos\left(\frac{2\,\pi\,t}{T} - q\,x\right) - (D'\,p + C'\,q)\sin\left(\frac{2\,\pi\,t}{T} - q\,x\right)\right] \quad \text{(7)}^{15}$$

in which, in addition to the notation of the paper, ξ = the displacement, or distance traveled by a particle of water in time, t, in feet; h = the height of the water surface at any section, x, referred to the elevation of mean tide, in feet; t = the time, in seconds, referred to the time of mean tide preceding high water; and x = the distance of any cross-section from the entrance to Cape Cod Bay, in feet.

The following are physical constants, determined by the conditions of the given particular problem: T = the time, in seconds, between successive high tides; e = the base of the natural system of logarithms; A = the area of the average cross-section of canal, in square feet; w = the unit weight of the liquid, in this case 64 lb per cu ft for sea water; X = the wetted perimeter of the average cross-section of canal, in feet; d = the reduced depth of the canal, in feet; that is, the depth of a rectangular section having the same area and top width as the average actual section, measured from the elevation of mean tide; and f_0 = a coefficient such that the friction force, F, per unit of surface is proportional to the nth power of the velocity so that $F = f_0\,v^n$, or $F = f_0$ for unit velocity.

For a locked canal with velocities of the order of 1 ft per sec, or less, f =0.00756 lb per sq ft; or,

$$f = \pm\,\frac{g\,f_0\,X}{w\,A} \quad \text{(8)}$$

$$p = \frac{\left(\frac{2\,\pi}{T}\right)}{\sqrt{2\,g\,d}}\sqrt{-1 + \sqrt{1 + \frac{f^2}{\left(\frac{2\,\pi}{T}\right)^2}}} \quad \text{(9)}$$

and,

$$q = \frac{\left(\frac{2\,\pi}{T}\right)}{\sqrt{2\,g\,d}}\sqrt{1 + \sqrt{1 + \frac{f^2}{\left(\frac{2\,\pi}{T}\right)^2}}} \quad \text{(10)}$$

The symbols, C, C', D, and D', are constants of integration to be determined from the terminal conditions of each particular case. For a locked canal they are determined as follows (see Figs. 12 and 13):

[15] The second minus (—) sign from the end of this equation is erroneously given as plus (+) in *Transactions*, Am. Soc. C. E., Vol. LXXXII (1918), p. 113, Equation (12).

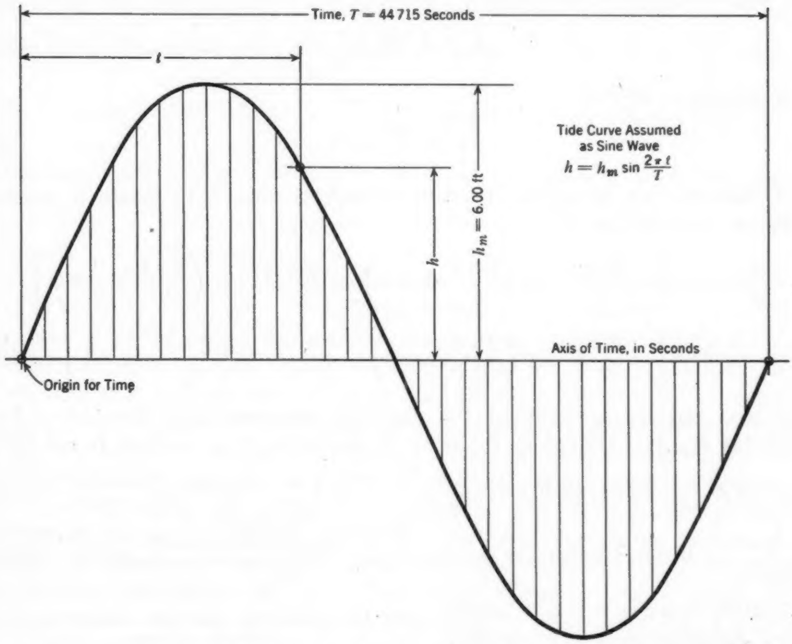


FIG. 12.—ASSUMED TIDE CURVE, CAPE COD BAY, AS A SINE WAVE, $h = h_m \sin \frac{2\pi t}{T}$.

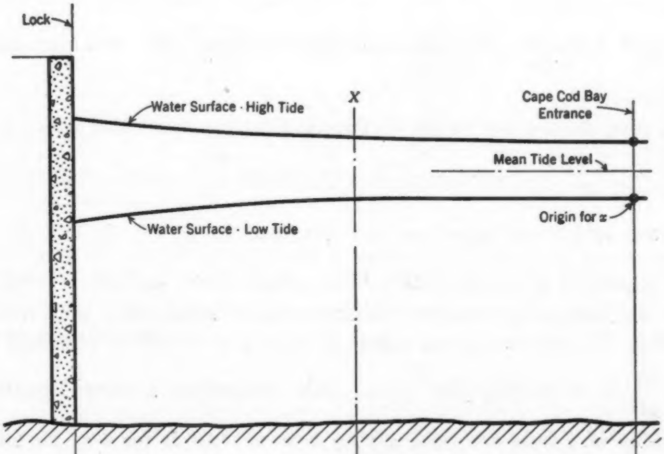


FIG. 13.—SCHEMATIC ELEVATION OF LOCKED CANAL.

When $x = 0$;

$$h = h_0 \sin \frac{2 \pi t}{T} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

and, when $x = 30\,000$,

$$v = \frac{d\xi}{dt} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

Differentiating Equation (6) with respect to time, t, to obtain a general equation for velocity:

$$\frac{d\xi}{dt} = e^{px} \left[- \frac{2\pi}{T} C \sin \left(\frac{2 \pi t}{T} + q x \right) + \frac{2\pi}{T} D \cos \left(\frac{2 \pi t}{T} + q x \right) \right]$$
$$+ e^{-px} \left[- \frac{2\pi}{T} C' \sin \left(\frac{2 \pi t}{T} - q x \right) - \frac{2\pi}{T} D' \cos \left(\frac{2 \pi t}{T} - q x \right) \right] \ldots (13)$$

The next step is to compute the physical constants (see Fig. 14). Let $X = 444$ ft; $A = 12\,250$ sq ft; $w = 64$ lb per cu ft; $g = 32.2$ ft per sec²; $d = 28.5$ ft; $L = 30\,000$ ft; $\frac{2\pi}{T} = \frac{2\pi}{44\,715} = 0.00014$ radian per sec; $f_0 = 0.00756$ lb per sq ft; $f = 0.000138$; $p = 0.00000208$; and $q = 0.00000507$.

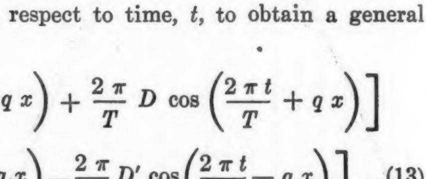


FIG. 14.—ASSUMED AVERAGE CROSS-SECTION. LOCKED CANAL.

Substituting the values of the foregoing physical constants and making $x = 30\,000$ and $\frac{d\xi}{dt} = 0$ in Equation (13); then setting $\frac{2 \pi t}{T}$ equal, successively, to 0 and $\frac{\pi}{2}$, two simultaneous equations are obtained involving C, C', D, and D'.

Next, substituting the values of the physical constants and making $x = 0$, $h = 6.00 \sin \frac{2 \pi t}{T}$ in Equation (7); then setting $\frac{2 \pi t}{T}$ equal, successively, to 0 and $\frac{\pi}{2}$, two additional equations are obtained in C, C', D, and D', giving $C = 1.77 \times 10^4$; $C' = -1.77 \times 10^4$; $D = -0.40 \times 10^4$; and $D' = -1.07 \times 10^4$.

With the foregoing constants of integration determined and substituted in Equation (7), the maximum value of h for $x = 30\,000$ is obtained by first placing $\frac{dh}{dt} = 0$, solving for t, and then computing h from Equation (7) for the value of t thus obtained, giving $h = \pm 6.10$ ft for $t = 20$ min after $\frac{T}{4}$ and $\frac{3T}{4}$. In other words, maximum high water and minimum low water at the lock site occur shortly after the time of occurrence of the corresponding levels at the entrance to Cape Cod Bay.

Similarly, with the constants of integration substituted in Equation (13), the maximum value of velocity, $\frac{d\xi}{dt}$, for $x = 0$, is obtained by placing $\frac{d^2\xi}{dt^2} = 0$, solving for t, and then computing $\frac{d\xi}{dt}$ from Equation (13) for the value of t thus obtained, giving $\frac{d\xi}{dt} = \pm\ 0.94$ ft per sec for $t = 0$ and $t = 22\,358.5$ sec. In other words, the maximum velocity at the entrance to Cape Cod Bay occurs close to the time of mid-tide.

Since the original objective in making the foregoing study was to determine the order of magnitude of the super-elevation and depression of tidal heights at the lock, rather than the refinement in the numerical result, slide-rule computations were used almost exclusively; and hence no claim is made to great accuracy in determining the critical times, which may be in error 10 min, or more. However, the following series of observations, brought to the attention of the writer by Lt. Col. Richard Park, Corps of Engineers, U. S. Army, then District Engineer at Boston, Mass., regarding a similar situation which occurred at the time when the Chesapeake and Delaware Canal was being lowered to sea level, tends to confirm the reliability of the method of analysis. The Delaware end of the Canal was opened to the tide for a distance of about three miles up to the lock at St. Georges. For a period of about two weeks this was as far as the tide could be propagated, and advantage was taken of the opportunity to set a gauge in the lock, so as to permit comparison of its readings with those of the Delaware River gauges. As shown in Table 11, the St. Georges ranges were perceptibly higher until the canal was cut through.

TABLE 11.—COMPARISON OF OPEN AND DEAD-END CONDITIONS, CHESAPEAKE AND DELAWARE CANAL

Location	High-water lunitidal interval, in hours	Height, in feet	Low-water lunitidal interval, in hours	Height, in feet	Range, in feet
Reedy Island	10.83	5.96	5.46	0.46	5.50
Reedy Point	11.19	5.90	5.59	0.53	5.37
St. Georges (February, 1927)	11.47	6.01	5.96	0.45	5.56
St. Georges (April, 1927)*	11.44	5.57	5.95	0.67	4.90

* After canal was cut through.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

STABLE CHANNELS IN ERODIBLE MATERIAL

Discussion

BY R. C. JOHNSON, M. AM. SOC. C. E.

R. C. JOHNSON,[3] M. AM. SOC. C. E. (by letter).[3a]—From a practical point of view it seems that the actual design of channels to prevent both erosion and silting must remain for some time to come largely a question of judgment. Although it is generally agreed that Kennedy's formula expresses a general law governing eroding velocities, the coefficient, C, will range through such wide variations for different soil types and other variable conditions, that the formula is of little value for use in the United States. The theoretical considerations, of course, are necessary for the designer as well as for those engaged in research, but such theory cannot supersede judgment based on a knowledge of local conditions.

The writer has had the opportunity of observing the erosion and silting in channels constructed by the United States Soil Conservation Service in the Piedmont Section of South Carolina during 1935. These channels were constructed chiefly for the purpose of drainage and have the following general characteristics: (1) The predominating type of soil is Cecil clay high in colloidal content (ranging as high as 46%, as determined by the Bouyoucos hydrometer method); (2) the flows in the waterways vary through wide limits, ranging from no flow to the maximum designed capacity; (3) practically all flows carry large quantities of silt; (4) the depth of flow seldom exceeds 2 ft; and (5) the channels range through a great variety of slopes and shapes. The ratio of width to depth ranges from about 3:1 to 10:1.

It is true that such channels do not come within the same classification as channels of more or less uniform flow; yet their consideration may have some related value. Observations were made repeatedly on a number of these channels, in sufficient numbers to lead to the following conclusions:

(*a*) Kennedy's formula for critical velocity was found to be definitely unsuited for design purposes, especially with a value of C even approximately as low as that recommended for Indian conditions;

NOTE.—The paper by E. W. Lane, M. Am. Soc. C. E., was published in November, 1935, *Proceedings*. This discussion is published in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[3] Assoc. Prof., Civ. Eng., School of Eng., Univ. of South Carolina, Columbia, S. C.

[3a] Received by the Secretary December 16, 1935.

(*b*) The roughness of the bottom plays a most important part in influencing bottom erosion for shallow depths of flow;

(*c*) Critical slopes exist beyond which no width-depth ratio will prevent erosion of the bottom, regardless of theoretical calculations. (This may be due chiefly to irregularities in the bottom which cannot possibly be avoided in construction of the channel under field conditions); and,

(*d*) Permissible canal velocities as published by Fred C. Scobey, M. Am. Soc. C. E., and the late Samuel Fortier, M. Am. Soc. C. E.[4], were found to be much better suited, as a rough guide, for general Piedmont conditions than the critical velocity given by Kennedy's formula.

At present, the problem of stable channels in erodible material is of major importance in soil-erosion control work. The problem is extremely complicated by the fact that the flow is most erratic and by the ever-present danger of excessive silting. It is believed, however, that it is possible to place the design on a more rational basis than can be done with existing knowledge of hydrodynamics and soil mechanics. The field for research along this line is practically unlimited.

[4] "Permissible Canal Velocities", *Transactions*, Am. Soc. C. E., Vol. 89 (1926), p. 940.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

TRUSS DEFLECTIONS: THE PANEL DEFLECTION METHOD

Discussion

By Messrs. William Bertwell, and Robert H. Hurlbutt

William Bertwell,[6] Assoc. M. Am. Soc. C. E. (by letter).[6a]—In calling attention to the process of computing truss deflections as the cumulative effect of movements of the component parts of the truss, this paper is interesting and valuable. The author states that the " 'panel deflection' method solves the problem of truss deflections in a simpler and more direct manner than other analytical methods." By the use of set formulas, however, the writer believes that the simplicity of the process is somewhat lessened.

By treating a single member at a time, rather than an entire panel, there results what may be termed the "geometric" method.[7] The computations for the author's Examples 1 and 2, presented in Tables 7 and 8, respectively, demonstrate that this basis of analysis is simpler.

The basis of the geometric method is: Assuming the part of the structure to the right of the member under consideration fixed in position, any change in length of the given member causes the part of the structure to the left to rotate about the center of moments for the given member. It remains only to determine the angular movement caused by the member, a geometric problem.

The writer believes that this one fundamental relationship is sufficient to solve all such problems simply and directly. Reducing the principle to a group of equations, or introducing supplementary derivations of one sort or another, inevitably diminishes understanding of the physical action of the structure. Keeping this one principle in mind, a glance at the line diagram of the structure is sufficient to establish the direction of movement imparted to any point by a given member. The figures are then set down as easily as a Williot diagram is drawn.

Every step of the geometric method appears in Tables 7 and 8; no formulas need be kept in mind. The figures for the chord members are obvious. The

Note.—The paper by Louis H. Shoemaker, M. Am. Soc. C. E., was published in November, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1936, by Messrs. David B. Hall, and E. Mirabelli.

[6] Associate Civ. Engr., U. S. Bureau of Public Roads, San Francisco, Calif.

[6a] Received by the Secretary December 19, 1935.

[7] *Engineering News-Record*, June 7, 1934, p. 746, and November 1, 1934, p. 565.

TABLE 7.—DEFLECTIONS OF A 180-FOOT SIMPLE TRUSS

Member	Change in length $\times \frac{30\,000}{12}$	Vertical component of movement at left end of member	Distance to center of moments (in panels)	Panel increment	DEFLECTIONS OF PANEL POINTS					
					0	1	2	4	3	6
3-5	−220.8	...	1.200	184.0	552.0	368.0	...	184.0	184.0	0
4-6	189.3	...	1.200	157.8	315.6	157.8	...	...	...	...
1-3	−222.5	...	1.184	188.0	376.0	188.0	...	...	...	...
0-4	369.0	...	1.033	357.2	357.2	...	...	...	...	...
3-6	131.0	170.3	...	0	170.3	170.3	...	170.3	170.3	...
3-4	14.4	...	7.200	2.0	−10.4	−12.4	...	−14.4	...	...
1-4	210.0	250.8	6.200	40.5	210.3	250.8	...	...	...	...
0-1	−286.0	396.7	...	...	396.7	...	...	...	...	...
1-2	159.3	...	...	...	...	...	−159.3	...	...	...
Total deflection from Point 6					2 367.7	1 122.5	...	339.9	354.3	0
Total deflection from Point 0					0	1 245.2	1 404.5	2 027.8	2 013.4	2 367.7
Total deflection from Point 0, in inches					0	0.50	0.56	0.81	0.81	0.95

effect of each diagonal web member has been determined by finding the vertical component of motion at the left end of the member and dividing this by the horizontal distance to the center of moments. The vertical component is equal to the change in length of the diagonal multiplied by the ratio of its length to the height of the truss at its right end.

TABLE 8.—INFLUENCE ORDINATES FOR HORIZONTAL REACTION OF 200-FOOT ARCH

Member	Change in length	Vertical component of movement at left end of member	Distance to center of moments (in panels)	Panel increment	Horizontal movement of Point 0	VERTICAL DEFLECTION OF PANEL POINTS				
						0	3	5	7	9
7-9	2.33	...	0.400	5.825	9.32	23.30	17.47	11.65	5.83	0
6-8	−2.51	...	0.498	5.040	10.08	15.12	10.08	5.04	...	...
5-7	2.14	...	0.500	4.280	6.42	12.84	8.56	4.28	...	...
4-6	−1.514	...	0.766	1.975	3.95	3.95	1.97	...	...	...
3-5	1.39	...	0.800	1.737	2.09	3.47	1.74	...	...	...
2-4	−0.857	...	1.163	0.737	1.47	0.74	...	...	...	...
1-3	0.50	...	1.300	0.385	0.27	0.38	...	...	...	...
0.2	−0.555	...	1.639	0.339	0.68	...	...	...	...	...
7-8	−1.38	3.71	5.000	0.742	1.48	5.94	5.19	4.45	3.71	...
7-6	0.42	...	5.000	0.084	0.17	0.67	0.59	0.50	...	...
5-6	−2.61	5.85	2.667	2.195	4.39	10.24	8.05	5.85	...	...
5-4	1.25	...	2.667	0.469	0.94	2.19	1.72	...	...	...
3-4	−2.62	4.19	2.600	1.613	3.22	5.80	4.19	...	...	...
3-2	1.66	...	2.600	0.638	1.28	2.30	...	...	...	...
1-2	−3.02	3.81	2.857	1.333	2.67	3.81	...	...	...	...
1-0	1.75	...	2.857	0.613	1.22	...	...	...	...	...
Total horizontal movement of Point 0					49.65 ×2 99.30					
Total deflection from Point 9						92.50	59.56	31.77	9.54	0
Total deflection from Point 0						0	32.94	60.73	82.96	92.50
Horizontal reaction $= \frac{\text{vertical deflection}}{99.30}$						...	0.332	0.612	0.836	0.932

ROBERT H. HURLBUTT,[8] JUN. AM. SOC. C. E. (by letter).[8a]—The method described in this interesting paper is based upon the area-moment principle as applied to a beam of varying moment of inertia. The same relation between the panel distortions and panel-point deflections of a truss may also be derived through inspection, as follows.

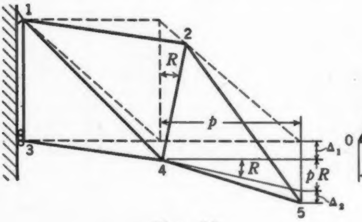


FIG. 11

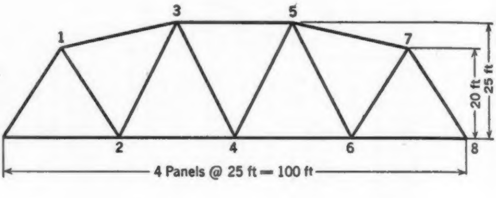


FIG. 12

In Fig. 11, a cantilever truss shown in a distorted position, the total deflection of Panel Point 5 is the sum of Δ_1, the vertical distortion of the first panel, plus R, the angular distortion of the first panel, multiplied by p, the length of the second panel, plus the vertical distortion, Δ_2, of the second panel.

TABLE 9.—DEFLECTIONS OF A 100-FOOT WARREN TRUSS

Member	Length, in inches	Area, in square inches	Stress, in thousands of pounds	Values of $\frac{SL}{A}$	Stress in member due to unit force, in pounds	Product of stress due to unit force and change of length	Stress in member due to unit couple, in pounds	Product of stress due to unit couple and change in length
0-1	283	20	—29.5	−416	−1.180	+493	0.0	
1-2	283	15	+23.0	+434	+1.180	+512	+0.000874	+0.379
0-2	300	15	+15.5	+310	−0.625	−194	−0.00416	−1.290
1-3	306	20	−28.5	−436	0.0		+0.00377	−1.645
2-3	336	8	−21.0	−884	0.0		−0.00083	+0.732
Δ_2						+811		
R_{2-3}								−1.824
2-3	336	8	−21.0	−884	−1.12	+990	0.0	
3-4	336	8	+28.0	+1.260	+1.12	+1 411	0.0	
2-4	300	15	+37.5	+750	−0.50	−375	−0.00333	−2.500
3-5	300	20	−50.0	−750	0.0		+0.00333	−2.500
4-5	336	8	−28.0	−1.260	0.0		0.0	
Δ_4						+2 026		
R_{4-5}								−5.0
4-5	336	8	—28.0	−1 260	−1.12	+1 411	0.0	
5-6	336	8	+47.0	+2 115	+1.12	+2 369	—0.00083	−1.755
4-6	300	15	+62.5	+1 250	—0.50	−625	—0.00333	−4.167
5-7	306	20	−85.0	−1 300	0.0		+0.00377	−4.910
6-7	283	15	+68.5	+1 290	0.0		+0.000874	+1.127
Δ_6						+3 155		
R_{6-7}								−9.705
6-7	288	15	+68.5	+1 290	−1.18	−1 522		
7-8	283	20	−88.5	−1 255	+1.18	−1 481		
6-8	300	15	+47.0	+940	−0.625	−587		
Δ_8						−3 590		

[8] Elec. Welder, Erection Dept., Chicago Bridge & Iron Works, Chicago, Ill.

[8a] Received by the Secretary December 20, 1935.

The panel-deflection method has advantages over the Maxwell-Mohr method for trusses with rectangular panels. The application is simple and direct, each step is easily visualized, and, most important, the deflections of all points are found from one set of calculations, as by the Williot diagram. However, a rotational correction corresponding to that provided by the Mohr rotation diagram, is necessary when dealing with a truss in which every member changes its line of action when the truss deflects.

Table 9 gives the calculations for Δ in a problem of this type, an unsymmetrically loaded Warren truss. Assuming that Member 0–1, Fig. 12, does not rotate, all deflections are found upward from Point 0. The distorted truss is then rotated downward to produce the effect of the rotation of Member 0–1. Thus, referring to Table 9 for values of Δ and R:

$$30\,000\ V_8 = 811 + 2\,026 + 3\,155$$

$$- 3\,590 - 300\ (3 \times 1.824 + 2 \times 5.0 + 9.705) = -5\,151$$

$$V_2 = \frac{1}{30\,000}\left(811 + \frac{5\,151}{4}\right) = +\ 0.070 \text{ in.}$$

$$V_4 = \frac{1}{30\,000}\left(811 + 2\,026 - 300 \times 1.824 + \frac{5\,151}{2}\right) = +\ 0.162 \text{ in.}$$

and,

$$V_6 = \frac{1}{30\,000}\left[811 + 2\,026 + 3\,155 - 300\,(2 \times 1.824 + 5.0)\right.$$

$$\left. + \frac{3 \times 5\,151}{4}\right] = +\ 0.242 \text{ in.}$$

The advantages of the panel-deflection method for the solution of this problem are not particularly great, due to the increased work involved in applying the unit forces and couples to the irregularly shaped panels.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

LATERAL PILE-LOADING TESTS

Discussion

BY AUGUST E. NIEDERHOFF, JUN. AM. SOC. C. E.

AUGUST E. NIEDERHOFF,[13] JUN. AM. SOC. C. E. (by letter).[13a]—The lateral pile-loading tests in this paper shed additional light on a subject that has long been obscure. Mr. Feagin's excellent record of the test data and the evident care and methodical procedure in gathering and presenting the information deserve commendation from the profession. It is noted that he is careful to state the limitations under which his findings were obtained, and invites discussion that will contribute to the general store of information on this subject.

Early in the period for designing locks and dams on the Upper Mississippi River, it was realized that specific knowledge of the resistance of round, wood, bearing piles to lateral loads was necessary. The design of these structures was based upon the assumption that all vertical loads were to be transmitted to the sub-strata by piles, leaving nothing for surface soil bearing, and it was believed to be just as logical an assumption to resist all lateral loads by piles and to omit all frictional resistance between the bottom of the structure and the ground surface. The design analysis, therefore, had to be based upon a certain allowable value and to arrive at this value a series of tests were inaugurated about December 19, 1933, at the proposed site of Lock No. 3 on the Mississippi River, immediately up stream from Red Wing, Minn.

This site was chosen for conducting the tests because borings indicated that soil conditions for a foundation were less desirable than at other Mississippi River lock and dam sites within the St. Paul (Minn.) District Office of the U. S. Corps of Engineers. A boring made where the tests were conducted is shown in Fig. 24. Both piles subjected to lateral loads were of red oak, 30 ft long, and with the tip diameters about 7 in. and 8 in., respectively, and ground-line diameters of about 12 in. each. The piles were driven about 28 ft through a mixture of silt and clay with a 3 000-lb steam hammer with

NOTE.—The paper by Lawrence B. Feagin, Assoc. M. Am. Soc. C. E., was published in November, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings*. as follows: November, 1935, by A. E. Cummings, Assoc. M. Am. Soc. C. E.; and January, 1936, by Messrs. J. C. Meem, and T. Kennard Thomson.

[13] Associate Engr., U. S. A. Engrs., St. Paul, Minn.

[13a] Received by the Secretary December 30, 1935.

a 33-in. stroke under 110-lb steam pressure. Allowing a penetration of 0.63 and 1.35 in. under the last blow, the indicated bearing capacity by the *Engineering News-Record* formula for Pile No. 2 was 10 tons and for Pile

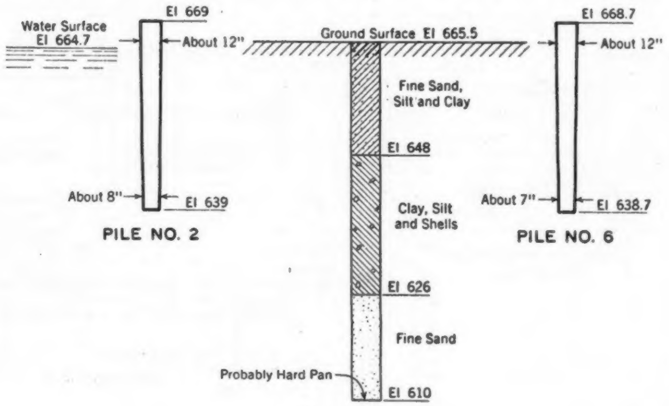


FIG. 24.

No. 6, 5 tons. This indicated bearing value is given only to show that the soil was not very compact. Some idea of the surface material can be obtained from the information that a man rolled up his sleeve and stuck his arm into the mud alongside one of the driven piles until it came above his elbow.

The equipment for the tests is shown in Fig. 25. The arrangement for testing Pile No. 1 at the upper end of the framework is shown in the plan view. Section *A–A* indicates the cable arrangement about the test pile. A load applied to the test pile by exerting a pull with a ratchet, *R*, at the other end of the lever beam, *A*, was measured on the 5 000-lb dynamometer to the left of, and attached to, the anchor pile, *X*. The lever beam, *A*, was pivoted on a case-hardened steel fulcrum supported by a frame work of 12 by 12-in. timbers. When sufficient load was applied by Ratchet *R*, the top of Test Pile No. 1 assumed an inclined position that was registered by an originally vertical flag attached firmly to the test specimen. To simulate the restraining action of the pile-butt embedded in concrete masonry, the flag was returned to a vertical position by pulling with Ratchet *S*. Transits checked the movement of the test specimen and kept the flag in a vertical position. The actual load on the pile, of course, was the difference in the readings of the two dynamometers on either side of the anchor pile, *X*, corrected by the ratio of the lever arms of Beams *A* and *B*. If the 5 000-lb gauge had been used to capacity, the maximum moment in Beams *A* and *B*, Fig. 25, would have been 1 620 000 in-lb and the extreme fiber stress, 27 500 lb per sq in., but because very few stiffeners had been provided the two beams showed a tendency toward diagonal buckling before they could be stressed to these values.

The nature of the equipment, therefore, limited the load that could be applied to the test specimen to 18 kips. Prior to running the tests, the soil around the test specimens had been frozen to a depth of approximately 6 in.

When testing Piles Nos. 2 and 6, this frozen ground was removed from around the pile for a distance of 4 in. It was believed that the remaining frozen ground would prevent the surrounding soil from bulging during the tests

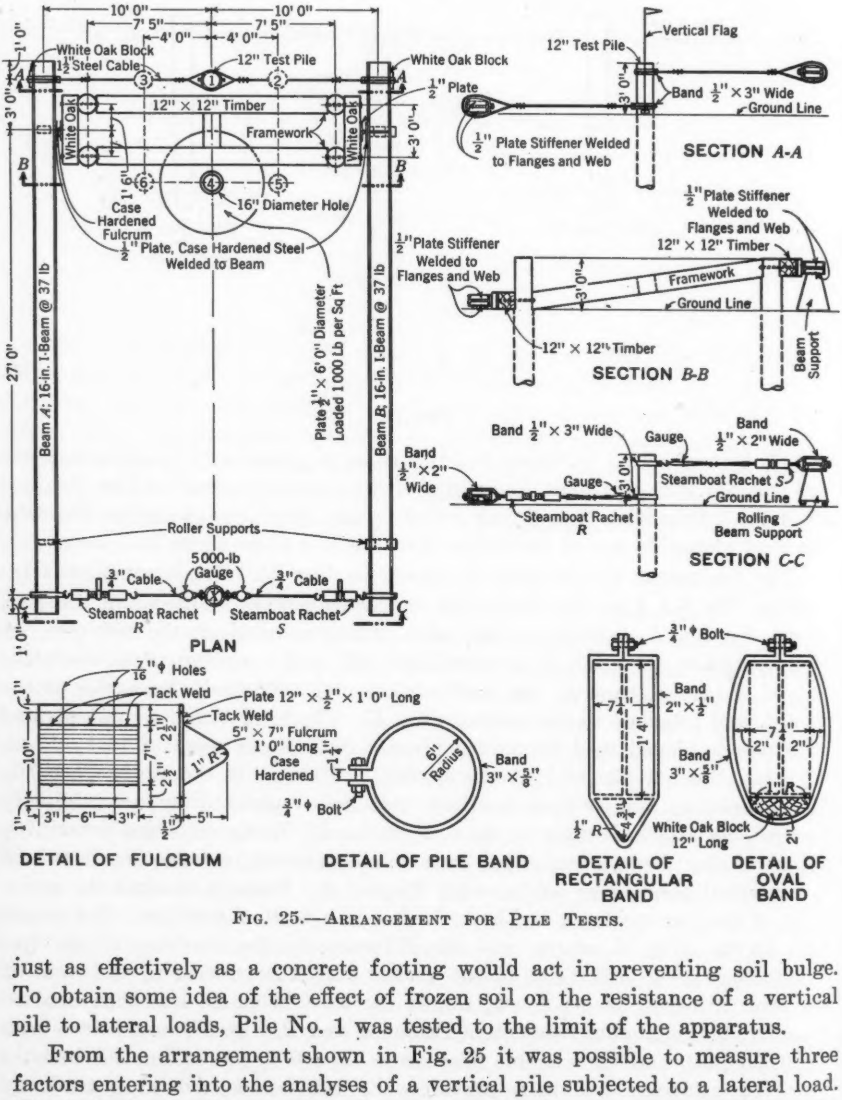


FIG. 25.—ARRANGEMENT FOR PILE TESTS.

just as effectively as a concrete footing would act in preventing soil bulge. To obtain some idea of the effect of frozen soil on the resistance of a vertical pile to lateral loads, Pile No. 1 was tested to the limit of the apparatus.

From the arrangement shown in Fig. 25 it was possible to measure three factors entering into the analyses of a vertical pile subjected to a lateral load. The deflection at the ground surface was measured to the nearest 0.02 in.; the applied load causing the deflection was measured on dynamometers graduated to 100 lb; and the moment in the pile produced by the arrangement of the two cables that simulated the restraining action of embedment of the butt in

masonry was measured by the known force exerted by each cable and the measured moment arm between the cable attachments on the test specimen. The recorded observations and the computed moment in the pile are presented in Table 4. A load of 17 kips was applied on Pile No. 6 (see Table

TABLE 4.—LATERAL PILE-LOADING TESTS, MISSISSIPPI RIVER LOCK No. 3, RED WING, MINN.

Test No.	Readings: Lower scale (1)	Readings: Upper scale (2)	Load, P_o, in pounds (3)	Moment,* M_o, in inch-pounds (4)	Deflection, in inches (5)
		(*a*) PILE No. 2			
1...	10 720	4 940	5 780	118 600	0.07
2...	18 340	7 020	11 320	168 400	0.13
3...	27 700	13 040	14 660	313 500	0.28
4...	25 850	11 540	14 310	277 000	0.26†
5...	27 050	12 250	14 800	294 500	0.30
6...	35 950	20 120	15 830	483 000	0.39
7...	30 900	12 830	18 070	308 000	0.36‡
		(*b*) PILE No. 6			
8...	4 165	2 310	1 855	55 400	0.01
9...	7 720	4 240	3 480	101 700	0.05
10...	16 400	9 110	7 290	218 500	0.15
11...	22 900	12 450	10 450	299 400	0.23
		(*b*) PILE No. 6 (*Continued*)			
12...	22 050	11 800	10 250	283 500	0.23†
13...	30 850	17 340	13 510	416 000	0.39
14...	34 500	19 670	14 830	472 000	0.46
15...	37 400	21 800	15 600	523 000	0.61
16...	36 300	19 270	17 030	462 500	0.63†
		(*c*) PILE No. 1			
17...	10 640	4 950	5 690	118 800	0.06
18...	20 530	10 900	9 630	261 600	0.07
19...	28 100	15 560	12 540	373 440	0.10
20...	36 800	20 740	16 060	497 760	0.16
21...	39 170	21 980	17 190	527 520	0.20

* 24-in. between clamps. † 5-min. intervals. ‡ Load released and re-applied.

4(*b*)) for a sustained 5-min interval. The pile continued to move under the influence of this load, thus substantiating the theory of plastic flow of the foundation soil.

Comparing the results of the Red Wing tests with the Alton, Ill., tests, it should be kept in mind that the former did not evaluate the effect of a lateral load applied to a group of piles. Recognizing this, it is remarkable that at Alton the maximum deflection of $\frac{9}{32}$ in. of a monolith when stressed to a lateral load per pile of 6.5 tons is comparable with the deflection of 0.28 and 0.39 in., respectively, of the piles at Red Wing when subjected to approximately an equivalent load. Among other considerations not evaluated by the Red Wing tests were: (*a*) The effect of confining soil around the pile in preventing or restraining plastic flow of foundation material; and (*b*) the effect of vertical load applied in conjunction with the lateral load.

Logically, it is believed that, to the time of applying loads in excess of 12 kips, the specimens assumed a center-line deflection curve of the same general shape as Fig. 17(*a*). Loads in excess of 12 kips probably caused plastic flow of the soil governed by the law of viscous flow in plastic materials and produced a center-line deflection curve described as fixed at the top against rotation, but showing considerable displacement, and at some depth, L, in the ground, showing no displacement, but some rotation. The bottom of the pile would then show a movement in a direction opposite to the top deflection. This center-line curve is believed to be simulated by the $\frac{1}{4}$-in. steel rod illustrated in Figs. 22 and 23.

The mathematical analysis of either of these curves involves certain assumptions that may be considerably in error. In the case of the Red Wing tests the lack of specific information and laboratory examination of the soil forestalls even an attempt at mathematical reasoning. If it were known definitely just what the volume change of the soil would be when subjected to increased pressure and the permeability and cohesion of the foundation material, a satisfactory attempt at induction might be tried, based primarily on soil mechanics.

On the basis of the Red Wing tests the District Engineer, at St. Paul, concluded that lateral load resistance of vertical round, wood piles is determined to a great extent by the nature and characteristics of the soil rather than by the flexural rigidity of the pile itself. There are certain limitations to this statement, of course, but for round, wood piles longer than 30 ft and with butt and tip dimensions in accordance with the standards specified in the A. S. T. M. specifications, the foregoing statement is believed to be valid. The extreme fiber stress on the pile that carried a maximum moment was 2 417 lb per sq in., and there was no apparent distress in the fibers. In making comparisons between the results of the tests and the behavior of actual vertical pile foundations when subjected to lateral loads, it might be mentioned that a lateral movement of a lock-wall occurred at Lock No. 5*A* on the Mississippi River when the computed horizontal load per round, wood pile did not exceed 3.5 tons. This lock-wall movement was approximately 9 in. in a foundation soil composed principally of sand with a thin clay stratum lying at considerable depth below the structure. It is believed that the combined effect of slippery clay and pile-driving in the vicinity of the wall produced this extraordinary lateral movement. The arresting fact is that construction conditions and the hydrostatic head that helped to move the wall at Lock No. 5*A* had been previously duplicated (although perhaps not quite so severely) at Lock No. 5. These conditions at Lock No. 5 had probably stressed the piles by a horizontal load of approximately 3 tons per wood pile. The foundation soil at Lock No. 5 is principally sand with some indications of gravel far below the lock-walls. The fact that no lateral deflection took place confirms the belief that soil investigation is of primary importance, and arbitrary "safe" values for lateral loads on piles are figments of the imagination.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

SEDIMENTATION IN QUIESCENT AND TURBULENT BASINS

Discussion

By Thomas R. Camp, M. Am. Soc. C. E.

Thomas R. Camp,[5] M. Am. Soc. C. E. (by letter).[5a]—In his paper Professor Slade revives interest in the development of a theory which is much needed for the design of sedimentation basins. Considerable attention has been given to this subject for a great many years among those concerned with water and sewage clarification. Despite this interest, it appears that an effective attack upon the problem in a rational way has not yet been made. The author's attempt to use a "distribution function" to describe the suspension is a step in the right direction. It is to be regretted, however, that so much of the paper is premised upon the validity of the assumptions made by the late Allen Hazen, M. Am. Soc. C. E., in his paper[6], published in 1904, setting forth what has become known as "Hazen's theory of sedimentation". Although Hazen's paper contained something of value toward the development of a rational theory, it fell far short of presenting a complete theory of sedimentation. Moreover, Hazen's theory contains so many invalid assumptions that it is of little practical use to sanitary engineers in design.

In order to discuss the present paper, it will be necessary to revive the discussion of Hazen's paper and also to refer to the paper entitled "Cleaning Water by Settlement"[7] by the late James Alexander Seddon, M. Am. Soc. C. E., which influenced Hazen considerably in the development of his theory.

As pointed out by Professor Slade, the settling in still water of a suspension of discrete particles, each of which settles at a constant velocity, will result in a faster clearing near the top of the basin than below it. The concentration of suspended matter will be reduced continuously during the settling period, but it will be greater near the bottom than near the top.

Note.—The paper by J. J. Slade, Jr., Esq., was published in December, 1935, *Proceedings*. This discussion is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion of the paper.

[5] Associate Prof. of San. Eng., Mass. Inst. Tech., Cambridge, Mass.

[5a] Received by the Secretary January 3, 1936.

[6] "On Sedimentation", *Transactions*, Am. Soc. C. E., Vol. LIII (1904), p. 45.

[7] *Journal*, Assoc. of Eng. Societies, 1889, p. 477.

Seddon anticipated this stratification, but found in experimenting with the sedimentation basins of the St. Louis (Mo.) Water-Works that the density was practically the same at depths of 2, 4, 6, and 8 ft. He considered both vertical mixing and coagulation in order to explain this inconsistency, and he concluded that the uniform density distribution was due to vertical mixing and not to flocculation. No scientific observations were made to support this conclusion, and it was drawn in spite of the fact that an increase in wind velocity over the basin from zero to 11 miles per hr produced no measurable effect upon the settling.

Hazen accepted the observations of Seddon that the concentration of suspended matter did not vary appreciably with depth, and he also accepted Seddon's conclusion that the cause was vertical mixing. No experimental data were presented by Hazen to substantiate either Seddon's observations, or his conclusions, and coagulation was definitely excluded in Hazen's theory as a contributing cause of uniform concentration.

It is well known that practically all the suspensions dealt with by sanitary engineers are composed of particles which are subject to flocculation during settling. It is not well known, however, that flocculation takes place to so great an extent in most cases as to invalidate the assumption that each particle settles at a constant velocity. Even with clay suspensions, such as Seddon was dealing with, flocculation takes place. In making wet mechanical analyses of clays by the hydrometer method, it is necessary to treat the suspensions with a peptizing agent, such as sodium silicate, in order to avoid the effects of coagulation.

The writer has been studying sedimentation both theoretically and experimentally during the past several years. In some of the experiments, the settling of ferric hydrate floc was studied in glass tubes equipped with sampling cocks at various depths below the water surface. Some samples, drawn simultaneously during settling from different depths, have shown substantially the same iron concentration. Other simultaneous samples have indicated slightly higher concentrations near the bottom, and still others have indicated slightly higher concentrations near the top. Similar analyses made in terms of suspended solids upon the suspension in the coagulation basin of the Cambridge (Mass.) filter plant, where alum is used for coagulation, have indicated a very small increase in concentration with depth. In the glass-tube experiments protected from temperature changes residual mixing currents could be observed for a period of only about 20 min after filling, but substantially uniform concentration throughout the depth was observed in some of the samples withdrawn as late as 7 hr after the water became quiet. Coalescence of particles during settling could be seen at any time during the period. Larger particles were observed to overtake smaller ones, enmesh them, and thus increase their size and hydraulic value.

In his discussion of Hazen's paper, the late Galen W. Pearsons, M. Am. Soc. C. E., describes[8] some experiments made under his direction preparatory to the design of the Kansas City (Mo.) settling basins. Three glass tubes,

[8] *Transactions*, Am. Soc. C. E., Vol. LIII (1904), p. 72.

3 in. in diameter and 5, 10, and 15 ft high, respectively, were filled with Missouri River water which was allowed to settle. To quote Mr. Pearsons,

"As it cleared gradually the writer was able to see particles descending near the bottom of the 15-ft tube; at times, something of the same could be discerned in the 10-ft tube, but with difficulty, and none at any time in the 5-ft tube.

"These particles, by their uniform shape, explained their origin and action; they were pear-shaped, or rather like little tadpoles swimming head down, the tails tapering to invisibility; plainly some larger particle by its quicker descent had overtaken and joined smaller ones, and, increasing by constant addition, had at last become visible, their motion near the bottom being so rapid that, if it had been uniform, but a few minutes would have been required for the whole descent."

The writer does not conclude as a result of these observations that the effect of coagulation will be the same for all suspensions or the same throughout the settling period for any one suspension. There is every reason to suppose that coagulation may be completely absent with certain granular suspensions, and may exhibit its effects in varying degrees with other suspensions. In some cases the concentration may be greater near the bottom and in others it may be less, all due to the degree of flocculation without any consideration of vertical mixing.

It is not the writer's contention that no vertical mixing takes place in sedimentation basins. Eddy currents are obvious in most basins of the continuous-flow type at the influent end. These eddy currents do not occupy any appreciable portion of the volume of the basin, however. Eddy currents are also produced by obstructions in basins, such as baffles, and by sludge-removal equipment. Convection currents due to temperature changes are sometimes present. Eddy currents so uniform in type and distribution throughout the basin as to produce the uniform distribution of suspended matter throughout the depth which has been observed, certainly do not exist for any considerable time in any basin. By means of dyes, the writer has observed the time required to damp the eddies produced by small submerged jets with a velocity of 1 ft per sec, and has found few of the eddies to last more than about 15 min.

It should be clearly understood that Hazen assumed the existence of vertical mixing in order to account for the uniform vertical distribution of suspended particles. There was no other object for this assumption. If vertical mixing is the cause of uniform concentration, the clarification process will be less rapid than in the case of still water. If flocculation is the cause, clarification will be more rapid than with discrete particles in still water. The importance of this assumption is thus apparent. That flocculation is the cause is evidenced again by the much greater speeds with which clays which have not been peptized will settle out in wet, mechanical, soil analysis. If the concentration of suspended matter in a basin is the same from top to bottom, but decreases as the time of settling increases, it is difficult to escape the conclusion that the removal is a function of the detention period and is thus not independent of the basin depth.

Having made the assumption of vertical mixing, Hazen makes the further assumption that all particles settle with the same velocity. All the equations

developed by Hazen are based upon these assumptions. Professor Slade attempts to apply a "distribution function" to Hazen's equations, in order to take into account the variation in settling velocities of the particles in the unsettled suspension. The application of the distribution function to Hazen's equations does not, as Professor Slade has assumed, maintain the uniform vertical distribution of sediment. B_a of the author's equations is the total quantity of suspended matter throughout the entire depth at the time, a. If a depth of $\frac{h}{2}$, for example, is taken at the end of the time, a, t' becomes $\frac{t}{2}$ and in the top half of the basin,

$$B'_a = B' e^{-\frac{2a}{t}} \qquad (37)$$

from Equation (27). t_M and t_m are one-half as much as for h, and from Equation (28),

$$B'_a = \frac{1}{2} \int_{t_m}^{t_M} \phi(t)\, e^{-\frac{2a}{t}}\, dt \qquad (38)$$

for the top half of the basin. The solution of this integral is considerably less than one-half the value of B_a from Equation (28). In other words, the density of sediment in the top half of the basin, as computed from the author's equations, is considerably less than the density in the bottom half. One must conclude that the author's attempt to apply a distribution function to Hazen's equations is a failure, in that it destroys the original assumption of uniform distribution of suspended matter from top to bottom.

Acceptance of the validity of Hazen's equations results in the conclusion that the percentage removal of sediment for a given discharge is a function of the surface area of the basin and independent of the depth. The introduction of the author's distribution function into Hazen's equations does not modify this conclusion. It is well to note that the author's equations apply to both fill and draw basins and continuous flow basins. As applied to continuous flow basins, the direction of flow is assumed to be horizontal and the velocity is assumed to be uniform over any cross-section of the basin. If the depth of the basin is reduced to $\frac{h}{2}$, for example, the settling period, a', will be equal to $\frac{a}{2}$; t' becomes $\frac{t}{2}$; and,

$$B'_a = B' e^{-\frac{a'}{t'}} = B' e^{-\frac{a}{t}} \qquad (39)$$

Since $B'_a = \frac{1}{2} B_a$ and $B' = \frac{1}{2} B$, the removal expressed as a ratio is,

$$\frac{B'_a}{B'} = \frac{B_a}{B} = e^{-\frac{a}{t}} \qquad (40)$$

The point is that Hazen's theory maintains that depth has no influence upon removal, a conclusion in support of which there is no experimental

evidence from basins clarifying water and sewage. Moreover, the theory does not maintain uniformity of concentration with depth when a distribution function is taken into account; and this, despite the fact that the theory was based primarily upon the assumption of uniform concentration and despite the fact that there is experimental evidence from actual basins in support of this assumption.

The selection of the experiments of Dilling and Pearse[4] upon the consolidation of sewage sludges in glass cylinders, by Professor Slade to illustrate his theory was an unfortunate one. In the first place, flocculation was present in these tests to a marked degree and was described fully by the experimenters. In the second place, the portion of the liquid occupied by the sludge was observed by noting the position of the top of the sludge blanket. This percentage of sludge by volume as reported by the experimenters is a very different thing from the percentage of sludge in suspension, the interpretation given by the author. After a brief time, none of the sludge is in suspension. All of it is supported by the bottom, and each particle thereafter settles at a diminishing rate. Consolidation is a vastly different phenomenon from free sedimentation. In sedimentation, the resistance to settlement is due entirely to the viscosity of the fluid. In consolidation, this resistance is augmented by the supporting power of the particles below.

The writer has made use of a distribution function in a theory of sedimentation of discrete particles, flocculation being excluded. Although this theory cannot be used for most of the suspensions dealt with by sanitary engineers, nevertheless it may be of value toward the development of a complete theory

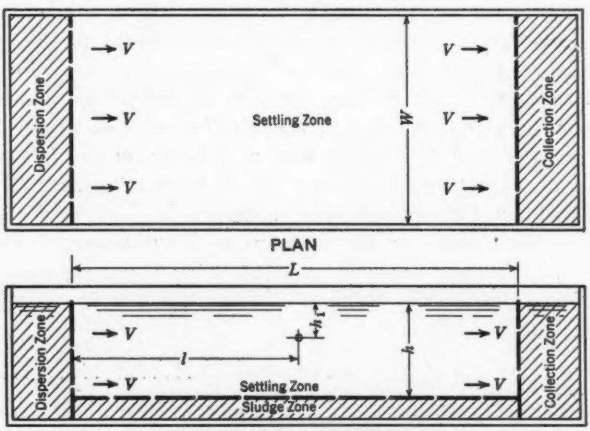


FIG. 10.—CONTINUOUS FLOW RECTANGULAR SETTLING BASIN.

including flocculation. The writer's distribution function has several advantages over that of Professor Slade, as will be apparent, and the theory has the advantage over that of Hazen in that mixing is specifically excluded.

[4] Rept. on Industrial Wastes from the Stockyards and Packingtown, in Chicago, by A. W. Dilling, Assoc. M. Am. Soc. C. E., and Langdon Pearse, M. Am. Soc. C. E., Vol. II, January, 1921 (The Sanitary Dist. of Chicago) pp. 140–141.

Both mixing and short-circuiting should be excluded as far as possible in an actual basin in the interest of efficiency, and a proper measure of the efficiency of a basin is the extent to which mixing and short-circuiting reduce the removal.

The notation used by Hazen and by Professor Slade will be utilized as far as is convenient. The theory will be presented for a "perfect", continuous-flow, rectangular basin. In such a basin it will be assumed that all settling takes place in a "settling zone", as shown in Fig. 10. As conceived by Hazen, a particle will be assumed to be removed if it reaches the bottom of the basin, or the top of the sludge blanket within the settling zone. Flow in the settling zone is assumed to be uniformly horizontal and at the same velocity, V, and direction throughout. The concentration of all suspended particles is assumed to be the same at all points in the plane of entrance to the settling zone.

In view of the assumptions made, the paths of the particles will be straight lines and all particles settling at the same velocity will move in parallel lines. The velocity and direction of settling of each particle will be the vector sum of its own hydraulic value and the velocity of flow of the liquid. Moreover the settling pattern will be the same in all longitudinal sections of the settling zone, and any longitudinal section may be considered as shown in Fig. 11, in order to study the removal.

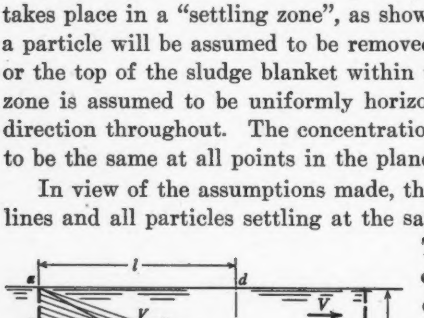


FIG. 11.—LONGITUDINAL SECTION, SHOWING PATHS OF PARTICLES IN SETTLING ZONE.

Consider any part of the length, l, of the settling zone. Particles which settle at the velocity, v, will move along paths parallel to the line, $a\ e$. The concentration of such particles above the line, $a\ e$, will be zero, and the concentration below this line will be the same as the initial concentration. All particles settling at the velocity, v, which enter below b, will settle within the shaded zone, $b\ c\ f$, and will be removed within the length, l. All particles of the settling velocity, v, entering above b, will cross the plane, $d\ f$, below e and, therefore, will not be removed in the length, l. If l is the length of the settling zone, such particles will pass out with the effluent. The removal, r, of particles of the settling velocity, v, in the distance, l, is, therefore, $\frac{b\,c}{h}$, or,

$$r = \frac{v\ a}{v\ t} = \frac{a}{t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (41)$$

in which v is less than v_0 and the removal, r, is expressed as a ratio; r is equal to $1 - \frac{B_a}{B}$. This is the same as Hazen's first formula and Equation (4) of Professor Slade's paper. If v is equal to or greater than v_0, all particles will be removed and r is equal to unity. Now, since $a = \frac{A\,h}{Q}$ and $t = \frac{h}{v}$, in which

A is the surface area of the basin; h, the depth; and Q, the discharge; the removal is, also,

$$r = \frac{A\ v}{Q} = \frac{v}{v_0} \qquad (42)$$

Equation (42) is Hazen's formula[9] and the writer's fundamental equation for discrete particles. From this equation, it may be noted that the removal of particles which settle at any constant velocity, v, less than v_0, is directly proportional to the surface area for any given discharge and is independent of the depth; or that the removal of such particles is inversely proportional to the "overflow rate", v_0. It may be noted that v_0 is equal to $\frac{Q}{A}$. The overflow rate is equivalent to a velocity, although it is most frequently stated in terms of the discharge per unit of surface area. All particles that settle at velocities equal to or in excess of v_0 will be removed. Hence, the removal from a suspension of discrete particles of varying hydraulic value will be a function of the surface area and independent of the depth of the basin.

An analysis of the suspension in terms of the settling velocities (that is, a distribution function) is required in order to predict removal for suspensions composed of discrete particles of different hydraulic values. Professor Slade has chosen to state his distribution function in terms of the time of settling of each particle from top to bottom. Although the function thus stated does describe the suspension, it is not independent of the depth of the basin. The writer prefers to state the function in terms of the hydraulic values of the particles. This practice has another advantage in that the settling velocity of very fine particles approaches zero, and zero may be used as a limit for integration. The time of settling of such particles approaches infinity, and, as the author has discovered, involves difficulties in the selection of an upper limit for integration.

The writer prefers to state the distribution function in terms of a mass curve, such as Fig. 3 of the paper, rather than as a rate curve such as Fig. 1. The distribution function thus stated will be analogous to a sieve analysis curve for sand. The advantages of this practice are that quantities are shown as linear dimensions rather than as areas; such a distribution function is more easily determined experimentally; and the computations for the removal of suspended matter are simplified very much.

Using Hazen's practice, the concentration of all the sediment in the unsettled or raw water will be designated as unity. Let P be the ratio of the initial concentration of particles which have hydraulic values of v, or less, to the total initial concentration; and P_0, the value of P corresponding to v_0. A typical distribution function of this type is shown by the curve of Fig. 12.

Since all particles which settle faster than v_0 will be 100% removed in the time, a, and the length, l, the removal of such particles in terms of the initial suspension is $1 - P_0$. The removal of particles having any settling

[9] *Transactions*, Am. Soc. C. E., Vol. LIII (1904), p. 55, Equation (7).

velocity, v, which is less than v_0 is $r = \frac{v}{v_0}$, and since the ratio of such particles to the total initial concentration is dP, the removal in terms of the total initial suspension is,

$$r\,dP = \frac{1}{v_0}\,v\,dP \qquad (43)$$

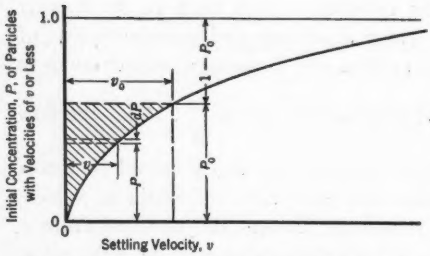


FIG. 12.—TYPICAL SETTLING VELOCITY ANALYSIS CURVE OF SUSPENSION.

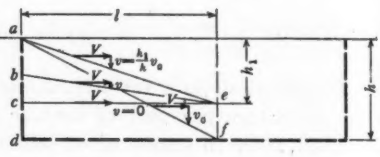


FIG. 13.—LONGITUDINAL SECTION OF SETTLING ZONE, SHOWING METHOD OF ESTIMATING CONCENTRATION AT A POINT.

To obtain the total removal of particles which settle at velocities of less than v_0, it is necessary to integrate Equation (43) between the limits of zero and P_0. The total removal, R, of all particles during the time, a, is therefore expressed as follows:

$$R = 1 - P_0 + \frac{1}{v_0}\int_0^{P_0} v\,dP \qquad (44)$$

If the equation of the analysis curve (that is, the distribution function) is known, the value of v in terms of P, or of dP in terms of dv, must be substituted in Equation (44), in order to solve the integral analytically. The expression under the integral sign, however, is the area on Fig. 12 shown shaded, and the last term of Equation (44) is the average vertical distance from the curve to the horizontal line at P_0. Hence, the removal may be measured graphically with sufficient accuracy by means of a planimeter, or with only a scale and slide-rule, without knowing the equation of the curve.

The concentration of suspended matter at any point in the settling zone, such as Point e on Fig. 13, may be estimated from the position of the point and the velocity-analysis curve of the original suspension.

Particles having any settling velocity, v, must cross the plane of entrance to the settling zone at b, in order to reach Point e. The concentration of such particles will be the same at Point e as at b, since they move along parallel paths, and the ratio to the total initial concentration, therefore, will be dP. Only particles that settle at velocities between zero and $\frac{h_1}{h}\,v_0$ can reach Point e. If the value of P on the velocity analysis curve which corresponds to the

velocity, $\frac{h_1}{h} v_0$, is designated by P_1, the concentration of suspended matter, X, at any point, e, in terms of initial concentration will be,

$$X = \int_0^{P_1} dP = P_1 \quad \text{.................... (45)}$$

The value of the integral is simply P_1, which may be read from the velocity analysis curve for any value of $\frac{h_1}{h} v_0$. The capital letter, X, has been used for the concentration at a point to distinguish from the average concentration in a vertical cross-section of the basin which Hazen denoted by the small letter, x. Now, since v_0 has one value at a given flow at any cross-section of the basin, it is obvious from Equation (45) that the density of suspended matter will be less near the top of the basin when settling discrete particles.

Equations (41) to (45) are also applicable to radial flow basins in settling discrete particles if the assumptions are made that flow is truly horizontal and radial at all points in the basin and that the velocity is the same at all points equi-distant from the center of the basin. The velocity, of course, will decrease, as the distance from the center increases, and all particles will settle along curved paths. Particles settling at the same velocity, however, will settle along parallel paths at any distance from the center of the tank.

If the frame of reference is transferred from the settling basin to the liquid moving within the settling zone, the velocity, V, becomes zero, and settling takes place as in still water. In other words, the settling which occurs in a vertical column of water as it moves through the basin is the same as that which occurs in an experimental tube, if all convection and eddy currents and wall effects are eliminated. Hence, experimental studies in tubes are an excellent means for determining the effects of coagulation in a "perfect" basin as defined previously. The difference between the observed concentration and that computed by means of Equation (45) is attributable to flocculation. It should be noted in this connection, that an experimental determination of the velocity analysis curve for discrete particles can be made readily; but no such curve exists for particles that flocculate since their velocities are increasing continuously.

In Fig. 10, the writer has shown a zone at the influent end of the basin for dispersion of the suspension uniformly over the cross-sectional area of the basin, and a similar zone at the effluent end for the collection of the clarified liquor uniformly over the cross-sectional area. It is very important in basin design to recognize the functions of these zones and to design them to perform their functions properly. This is an hydraulic problem which has not been effectively solved in the design of most existing basins. The penalties for inadequate solutions are vertical currents and short-circuiting, the latter being, undoubtedly, one of the major causes of basin inefficiency.

In the development of his theory, Hazen assumed the existence of vertical planes in the basin perpendicular to the direction of flow at which instantaneous decreases in concentration of sediment take place. To these planes,

he gave the name, "baffles", and thereafter stated that real baffles such as vertical walls or partitions across the basin so as to divide it partially into two or more sections in series were equivalent in their function to his imaginary baffles. Such is not the case, and it is difficult to conceive of any physical device other than a fine screen or a filter placed in a vertical cross-section of the basin, which will produce a sudden decrease in concentration. All baffles across the direction of flow within the settling zone produce vertical currents which tend to carry particles upward, produce higher velocities, and produce eddy currents. They are objectionable, therefore, within the zone of settling.

Many reports have been published which indicate improvements in settling efficiency due to baffling. The increases in efficiency were doubtless observed, but the cause had nothing to do with Hazen's theory or the effectiveness of baffles generally in increasing basin efficiency. The cause was an inadvertent lessening by the baffles of the degree of short-circuiting previously present, and a corresponding increase in the flowing-through period. The cause was not that the basin was divided into two or more parts by baffling, but that more of the volume of the basin was made effective for settling. The increase in efficiency could have been made more effectively by improving the design of both influent and effluent zones with the definite aim of minimizing short-circuiting.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

THE STRESS FUNCTION AND PHOTO-ELASTICITY APPLIED TO DAMS

Discussion

By Elmer O. Bergman, Assoc. M. Am. Soc. C. E.

Elmer O. Bergman,[35] Assoc. M. Am. Soc. C. E. (by letter).[35a]—The treatment of the stresses due to the weight of the structure, as proposed in this paper, is a particularly valuable contribution to the subject. The author is to be commended, furthermore, for his able presentation of the Airy stress function and its application to the analysis of civil engineering structures.

In the past the Airy stress function has been applied mainly to the solution of problems of interest to the mechanical engineer in which the stresses due to weight can be neglected in comparison with those set up by the applied loads. Another important point of difference between the problem treated by the author and those dealt with in the literature of elasticity lies in the disparity of cost between a dam and the ordinary structural and machine elements. Because the solution of all but the simplest problems of elasticity is involved and laborious, an effort is made to obtain a design formula that is relatively easy to apply and is on the side of safety. This necessarily results in the use of more material which, in a large dam, might result in a considerable increase in cost. The author has developed an approximate solution in Application IV, but he recommends that its use be limited to preliminary work.

Any variable which is a function of two independent space variables can be represented by a surface. The surface represented by the Airy stress function is called the Airy surface. The curvatures and twists of such a surface using a small scale for F are given by Equations (1) and hence afford a geometrical representation of the stresses. It should be noted that, in this interpretation, the curvature of the surface along any section is related to the normal stress acting across that section. Thus, the curvature at a point in the direction parallel to x represents the stress, σ_y, at that point.

Note.—The paper by John H. A. Brahtz, Esq., was published in September, 1935, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1935, by I. K. Silverman, Jun. Am. Soc. C. E.; December, 1935, by Fred L. Plummer, Assoc. M. Am. Soc. C. E.; and January, 1936 by Messrs. A. G. Solakian and Lars R. Jorgensen.

[35] Assoc. Prof. of Civ. Eng., Univ. of Colorado, Boulder, Colo.

[35a] Received by the Secretary January 3, 1936.

The condition that Equations (6) reduce to Equations (7) is that the x, y plane be tangent to the Airy surface at the origin. When no body forces are acting, the value of the Airy stress function, F, at any point (by Equation $7(c)$), represents the moment of the forces between the origin and the point about the point. Thus, if the origin is taken as in Fig. 1, the moment at B is the moment about B of the internal and external forces acting along any line connecting the origin with B.

The Airy surface, therefore, is a two-dimensional analogue of the familiar bending moment diagram. The slope of the tangent line to the surface at any point represents the sum from the origin to the point of the forces perpendicular to the tangent line. Thus, the Airy surface furnishes a helpful means of visualizing the moments, shears, and stresses in the slab.

The writer has been connected with the testing and analysis of several models of dams constructed of a compound of commercial building plaster and diatomaceous earth, a material very similar in action to concrete, but having a much lower modulus of elasticity. The models consisted of thin slices from the central part of the dam where they can be considered as being in a state of plane strain. As tested, the models were in a state of plane stress. Under these conditions there is direct correspondence between the stresses in the model and prototype, but not between the displacements and the strains. (For the relations between displacements, see the author's discussion relating to Equations (9) to (12).) The strains due to dead load and live load were measured on four intersecting gauge lines at points in the dam and its foundation with optical strain-gauges.

Vertical, horizontal, shearing, and principal stresses were computed from these strains and the elastic constants of the material. The results of these tests can not be compared directly with those of the author because they were not on the same section, but they show the same trend of action as indicated in Figs. 6 and 7, except that, due to plastic flow in the material of the model under high stress, the high concentration of stress at the heel and toe of the dam did not appear.